Math 444 - Contour Integrals **Name:** ______________________

I just showed that

$$\int_0^{2\pi} \frac{\sin^2\theta}{5+4\cos\theta}\,d\theta = \frac{1}{4i}\oint_{|z|=1} \frac{(z^2-1)^2}{2z^4+5z^3+2z^2}\,dz.$$

In the next few problems we will calculate the value of this definite integral.

1. Factor the polynomial $2z^4+5z^3+2z^2$ into a product of linear factors.

2. Calculate $\oint_C \frac{(z^2-1)^2}{2z^4+5z^3+2z^2}\,dz$ when C is a very small circle around the origin (radius is much smaller than 1/2).

3. Calculate $\oint_{|z|=1} \frac{(z^2-1)^2}{2z^4+5z^3+2z^2}\,dz$.

4. Use the answer to the previous problem to find $\int_0^{2\pi} \frac{\sin^2\theta}{5+4\cos\theta}\,d\theta$.

5. Find all four roots of $z^4 + 1$ and use them to completely factor $z^4 + 1$.

6. Let C_1 be the half-circle $\{Re^{i\theta} : 0 \le \theta \le \pi\}$. Show that $\lim_{R\to\infty} \int_{C_1} \frac{1}{z^4+1}\, dz = 0$.

7. Evaluate $\int_{-\infty}^{\infty} \frac{dx}{x^4+1}$.

8. (**Extra Credit**) Evaluate $\int_{-\infty}^{\infty} \frac{\cos x}{x^4+1}\, dx$.