Math 444 - Entire Functions **Name:** ______________________

1. Suppose that f is entire and there exists $M > 0$ such that $|f(z)| \geq M$ for all $z \in \mathbb{C}$. Use Liouville's theorem to prove that f is constant.

2. Suppose that f is entire and $\operatorname{Re} f$ is bounded. Prove that f must be constant. Hint: Consider the function $\exp(f(z))$.

3. One of the roots of the polynomial $p(z) = z^3 - 6z + 20i$ is $z = 2i$. Factor $p(z)$ into a product of linear factors. Hint: Use polynomial long division to remove the factor $(z - 2i)$ first.

4. Suppose that f is entire and $|f(z)| \leq |z|^p$ for all $z \in \mathbb{C}$ where p is a fixed positive number less than 1. Prove that $f(w) = 0$ for all $w \in \mathbb{C}$. Hint: Use the Cauchy formula for derivatives to estimate $f'(w)$ by calculating

$$f'(w) = \frac{1}{2\pi i} \oint_{C_R} \frac{f(z)}{(z-w)^2}\, dz$$

where C_R is a circle of radius R around w and the radius R is allowed to be arbitrarily large.