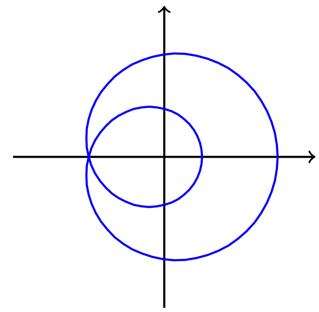
Math 444 - Cauchy's Integral Formula **Name:** ______________________

1. Let $\gamma(t) = 2e^{2it} - e^{it}$, $0 \leq t \leq 2\pi$. This path loops around the origin twice as shown below. Calculate $\int_\gamma \frac{dz}{z}$ for this path. Hint: You can make it easier if you break the path into two simple closed curves, an inner one and an outer one, then apply the Cauchy Integral Formula.



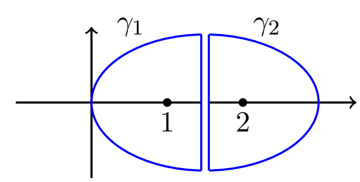
2. Let γ be the ellipse $|z-1| + |z-2| = 3$. Use a partial fraction decomposition to calculate

$$\int_\gamma \frac{z}{(z-1)(z-2)}\, dz$$

3. Compute $\oint_C \frac{e^z}{z(z-3)}\, dz$ where C is the circle of radius 2 around $z = 3$.

4. Compute $\oint_{|z|=4} \frac{e^z}{z(z-3)}\, dz$.

5. What if you calculate the integral in problem 2 by splitting the elliptical path into a sum of two separate integrals along positively oriented paths γ_1 and γ_2 as shown in the figure below? Find the values of $\int_{\gamma_1} \frac{z}{(z-1)(z-2)}\, dz$ and $\int_{\gamma_2} \frac{z}{(z-1)(z-2)}\, dz$. Check to see if the sum of these two integrals is the same as the integral in problem 2.



6. Calculate $\oint_{|z|=2} \frac{e^z}{z^2+1}\, dz$.

7. If $p(z)$ is a polynomial with no roots on the unit circle, then prove that the integral of $\frac{1}{z}$ on the path $\gamma(t) = p(e^{it})$ with $0 \le t \le 2\pi$ is the same as $\oint_C \frac{p'(z)}{p(z)}\, dz$ where C is the unit circle.