Math 444 - Taylor Series **Name:** ____________________

Find Taylor series for the following functions with the given centers.

1. $f(x) = \dfrac{\sin(z^3)}{z^2}$ centered at 0.

2. $f(x) = \dfrac{1}{z + 2i}$ centered at 0.

3. $f(x) = \text{Log}(z + 2i)$ centered at 0. Hint integrate the last Taylor series!

4. e^z centered at $c = \pi i$. For this one you will have to use the Taylor series formula.

5. Suppose that $f(z) = (z-a)^m g(z)$ where g is holomorphic in some disk around a and $g(a) \neq 0$ in that disk. Show that

$$\frac{f'(z)}{f(z)} = \frac{g'(z)}{g(z)} + \frac{m}{z-a}.$$

6. If γ is a closed, piecewise smooth curve in the disk where the function g in the last problem is holomorphic, then the path $f(\gamma(t))$ has winding number

$$\frac{1}{2\pi i} \int_\gamma \frac{f'(z)}{f(z)}\, dz.$$

Show that this winding number must equal m if γ is a simple curve around a.