Math 444 - Using Sage

Name: ______________________

Sage is a free and open source computer algebra system built on Python. To try Sage out, you can use the SageMathCell `https://sagecell.sagemath.org/`. If you want to use Sage more, you can download it on your own computer, or you can sign up for a CoCalc account. For this assignment, just enter the URL for above into your browser.

Defining Symbolic Variables

Sage is a computer algebra system, so it can simplify, expand, and factor algebraic expressions. To enter an expression with variables, you first have to tell Sage which symbols are variables. I like to use x, y, t for real variables and w, z for complex variables. Here is how to do this in Sage.

```
var('x,y,t',domain='real')
w, z = symbols('w,z')
```

Notice that you don't need to tell Sage that w and z are complex since that is the default assumption.

Complex Numbers in Sage

In Python, complex numbers are entered using the symbol $1j$. Sage is built on Python, so you can use $1j$, but you can also use a capital or lower case i symbol to represent $\sqrt{-1}$. So you can enter complex numbers like this `3+4j` or like this `3+4*i`. There is a difference between the two formulas. If you do a calculation with `3+4j`, Sage will give an (imprecise) decimal result. If you use `3+4*i`, then Sage tries to give an exact answer. Try this code and write down the output:

```
print(1/(3+4j))
print(1/(3+4*I))
```

Trying Things Out

Try the following commands. What happens with each one?

```
print(expand((z-2*I)*(z+3*I)))
print(diff(exp(z)/sin(z),z))
```

Substituting Values

In Sage you can input values into symbolic expressions. For example, if `f = z^2`, then you can plug $z = 5$ into f by doing the following:

```
f=z^2
print(f(5))
print(f(z=5)) # This is the preferred method for substituting values
```

Lists and List Comprehensions

To get all one hundred $100th$-roots of unity, you can use a list comprehension. In Python (and Sage) a list is a collection of values in brackets. A list comprehension like the one below is a quick way to get a list of values.

```
[exp(2*pi*i/100*k) for k in range(100)]
```

Plotting Points and Curves in $\mathbb{C}$

You can plot the list of 100th roots of unity using the `list_plot` function:

```
list_plot([exp(2*pi*i/100*k) for k in range(100)],aspect_ratio=1)
# You can also change these default arguments: color='blue', xmin=-1, xmax=1, ymin=-1, ymax=1
```

You can plot a curve using the `parametric_plot()` function:

```
f = exp(i*t)
parametric_plot( (real(f),imag(f)), (t,0,pi/2))
```

Exercises

1. Use Sage to find the derivative of $f(z) = \dfrac{z^2 + \cos z}{e^z}$.

2. Do the following calculations with Sage:

   (a) Simplify $\dfrac{1}{(5+12i)^2}$

   (b) Compute $(3+4i)^4$

3. Find the sum of the imaginary parts of the complex numbers $\{(1+i)^k : k \in \mathbb{N} \text{ and } 1 \leq k \leq 10\}$.
   Hint: Make sure to include $k = 10$ in your sum.

4. Make a plot showing the points in the set $\{(1 + \frac{i}{100})^k : k \in \mathbb{N} \text{ and } 0 \leq k < 1000\}$.