

Homework 19

1. Write the 4×4 transformation matrix for the translation where $\Delta x = 5$, $\Delta y = 8$, and $\Delta z = -3$.
2. Multiply the following two translation matrices and verify that the resulting matrix represents the composition of the translations.

$$A = \begin{pmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 4 \\ 0 & 0 & 1 & 6 \\ 0 & 0 & 0 & 1 \end{pmatrix} \text{ and } B = \begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 5 \\ 0 & 0 & 0 & 1 \end{pmatrix}.$$

3. Write the 4×4 transformation matrix for the scaling where $s_x = 2$, $s_y = 4$, and $s_z = \frac{1}{2}$.
4. Multiply the following two scaling matrices and verify that the resulting matrix represents the composition of the scalings.

$$A = \begin{pmatrix} 3 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 5 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \text{ and } B = \begin{pmatrix} 5 & 0 & 0 & 0 \\ 0 & 4 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}.$$

5. Write the 4×4 transformation matrix for the rotation about the x -axis through an angle $\theta = 45^\circ$.
6. The following matrices represent rotations about the z -axis of 45° and 90° , respectively. Multiply them together and verify that the resulting matrix represents a rotation of 135° about the z -axis.

$$A = \begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 & 0 \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \text{ and } B = \begin{pmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}.$$

7. Use matrices to verify that the composition of the two scalings ($s_x = -1$, $s_y = 1$, $s_z = 1$) and ($s_x = 1$, $s_y = -1$, $s_z = 1$) is the same as a rotation of 180° about the z -axis. [The first scaling represents a *reflection* in the x -axis and the second scaling represents a *reflection* in the y -axis. In general, the composition of two reflections is a rotation!]