Coms 331 **Arbitrary Rotations** **Fall 2007**

Homework 21

1. In class we found that if we rotate about the axis $(\frac{1}{3}, \frac{2}{3}, \frac{2}{3})$ through an angle of $90°$, then the rotation matrix is

$$\begin{pmatrix} 1/9 & -4/9 & 8/9 & 0 \\ 8/9 & 4/9 & 1/9 & 0 \\ -4/9 & 7/9 & 4/9 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}.$$

   Find the images of the points $i = (1, 0, 0, 1), j = (0, 1, 0, 1)$, and $k = (0, 0, 1, 1)$ under this rotation.

2. Given that the inverse of a rotation matrix is the same as its transpose, write the inverse of the rotation matrix in the first exercise. What do you notice about the coordinates of these images?

3. Find the images of the points i, j, and k under the inverse rotation in exercise 2. These images are the *pre-images* of the points i, j, and k under the original rotation. What do you notice about the coordinates of these pre-images?

4. Let the axis of rotation be given by the vector $\mathbf{v} = (\frac{2}{3}, \frac{1}{3}, \frac{2}{3})$ and let the angle or rotation be $180°$. Find the angle β that was discussed in class and then find $\cos\beta$ and $\sin\beta$.

5. Find the matrix $\mathbf{R}_y(\beta)$.

6. Find the vector $\mathbf{v}' = \mathbf{R}_y(\beta)\mathbf{v}$.

7. Find the angle γ that was discussed in class and then find $\cos\gamma$ and $\sin\gamma$.

8. Find the matrix $\mathbf{R}_z(\gamma)$.

9. Find the vector $\mathbf{v}'' = \mathbf{R}_z(\gamma)\mathbf{v}'$ and verify that it lies along the positive x-axis.

10. Find the matrix $\mathbf{R}_x(\alpha)$, where $\alpha = 180°$, the original angle of rotation.

11. Find the inverses of the matrices $\mathbf{R}_y(\beta)$ and $\mathbf{R}_z(\gamma)$. Remember that the inverse of a rotation matrix is the same as its transpose.

12. Find the product $\mathbf{M} = \mathbf{R}_y(\beta)^{-1}\mathbf{R}_z(\gamma)^{-1}\mathbf{R}_x(\alpha)\mathbf{R}_z(\gamma)\mathbf{R}_y(\beta)$.

13. Since a rotation of $180°$ applied twice will return all points to their original positions, verify that the square of the matrix $\mathbf{M}$ is the identity matrix.

14. Any point lying on the axis of rotation, given by the vector $\mathbf{v} = (\frac{2}{3}, \frac{1}{3}, \frac{2}{3})$, is of the form $P = (\frac{2t}{3}, \frac{t}{3}, \frac{2t}{3})$. Verify that any such point is mapped to itself by $\mathbf{M}$. That is, verify that $\mathbf{M}P = P$.