Coms 331 Meshes Generated by Functions Fall 2007

Homework 25

1. Let $f(x, y) = 3x^2 + 4xy + 5y^2$. Find $\partial f/\partial x$ and $\partial f/\partial y$.

2. For the function $f(x, y)$ in the previous exercise, find the tangent vectors $\mathbf{u} = (1, 0, \partial f/\partial x)$ and $\mathbf{v} = (0, 1, \partial f/\partial y)$.

3. Use $\mathbf{u}$ and $\mathbf{v}$ of the previous exercise to find the unit normal vector $\mathbf{n}$ to the surface $z = f(x, y)$.

4. The equation $z = 1 - \sqrt{x^2 + y^2}, -1 \le x \le 1, -1 \le y \le 1, 0 \le z \le 1$, describes the surface of a cone whose base is the unit circle in the xy-plane and whose vertex is at $(0, 0, 1)$. Find the unit normal vector $\mathbf{n}$ to this surface.

5. The equation $z = \cos x + \sin y$ describes a wavy surface. Find the unit normal vector $\mathbf{n}$ to this surface.

6. We have calculated $\mathbf{N}$ as $\mathbf{u} \times \mathbf{v}$ and then normalized to get $\mathbf{n}$. What would happen if we calculated $\mathbf{N}$ as $\mathbf{v} \times \mathbf{u}$ and then normalized? Would there be a difference? Would it matter?

7. Let $z = \sqrt{1 - x^2 - y^2}, -1 \le x \le 1, -1 \le y \le 1$, the upper hemisphere of the unit sphere. Let $m = 4$ and $n = 2$ to partion the surface into eight quadrilaterals. That is, partition $[-1, 1]$ on the x-axis into $m = 4$ subintervals (5 points) and partition $[-1, 1]$ on the y-axis into $n = 4$ subintervals (5 points). Compute the coordinates of the 25 grid points in the xy-plane.

8. Compute as many of the 3D mesh points as possible that correspond to the 25 grid points found in the previous exercise.

9. In the previous problem, are the grid points evenly spaced over the surface? What exactly would the grid quadrilaterals be? Suppose we used triangles instead of quadrilaterals (*triangulation*). Would the results be better? Does this partition appear to be satisfactory for the purpose of rendering a hemisphere?

10. Use your imagination. If we used a partition of 40 points in both the x and y directions and then created a mesh of quadrilaterals on the hemisphere, what would the lower boundary of the mesh look like?

11. For the surface $z = \sqrt{1 - x^2 - y^2}$, find the formula for the unit normal vector $\mathbf{n}$.

12. Use the results of the previous exercises to calculate the unit normals for each of the mesh points of the hemisphere.