Coms 331 **Parameterized Meshes** **Fall 2007**

Homework 26

1. Let a surface be defined by $x = s+1$, $y = t-s$, and $z = 2t$, $0 \le s \le 1, 0 \le t \le 1$. Verify that this is the surface that is defined explicitly by $z = 2(x + y - 1)$.

2. In the previous exercise, let $P(s,t) = (x, y, z)$ and find $\partial P/\partial s$ and $\partial P/\partial t$.

3. Compute $\mathbf{N} = (\partial P/\partial s) \times (\partial P/\partial t)$ and normalize it to a unit vector $\mathbf{n}$.

4. Verify that the normal found in the previous problem is the same as the normal that we would find using the partials $\partial z/\partial x$ and $\partial z/\partial y$ and the equation $z = 2(x + y - 1)$.

5. The parametric equations $x = t\cos s, y = t\sin s, z = 1-t$, with $0 \le s \le 2\pi, 0 \le t \le 1$, describes the cone whose base is the unit circle in the xy-plane and whose vertex is at $(0, 0, 1)$. Find the unit normal vector $\mathbf{n}$ to this surface.

6. The parametric equations

$$\begin{aligned} x &= \cos s \cos t \\ y &= \sin s \cos t \\ z &= \sin t, \end{aligned}$$

where $0 \le s \le 2\pi$ and $0 \le t \le \frac{\pi}{2}$, describes the upper hemisphere of the unit sphere. Partition this surface by letting $m = 4$ and $n = 2$. That is, partition the s interval $[0, 2\pi]$ into four parts (5 points) and partition the t interval $[0, \frac{\pi}{2}]$ into 2 parts (3 points). Calculate the x-, y-, and z-coordinates of each grid point.

7. In the previous problem, are the grid points evenly spaced over the surface? What exactly would the grid quadrilaterals be? Does this partition appear to be satisfactory for the purpose of rendering a hemisphere?

8. Use your imagination. If we used a partition of 20 points in both the x and y directions and then created a mesh of quadrilaterals on the hemisphere, what would the lower boundary of the mesh look like?

9. Use the parametric equations for the hemisphere to find a formula for the unit normal vector $\mathbf{n}$.

10. By multiplying z by 2, we may stretch the sphere into an ellipsoid. Let $x = \cos s \cos t, y = \sin s \cos t$, and $z = 2\sin t$. Find the unit normal vector $\mathbf{n}$ for this ellipsoid.

11. What was the effect on the components of $\mathbf{n}$ of doubling the z coordinate?