Coms 331 **Preserving Normals** **Fall 2007**

Homework 28

1. On a sheet of graph paper, using 5 squares to represent one unit, draw a line segment from $(0,1)$ to $(1,0)$. Then draw a normal to the line. What is the slope of the line segment and what is the slope of the normal?

2. Make a separate drawing of a line segment from $(0,1)$ to $(2,0)$, i.e., the original segment stretched in the x direction by a factor of 2. Draw a normal to that line. What is the slope of the line segment and what is the slope of the normal? How do they compare to the slopes in the previous exercise?

3. For the matrices
$$\mathbf{A} = \begin{pmatrix} 1 & 3 \\ 5 & 4 \end{pmatrix} \text{ and } \mathbf{B} = \begin{pmatrix} 0 & 2 \\ 3 & 5 \end{pmatrix}$$
verify that $\mathbf{B}^{\mathrm{T}}\mathbf{A}^{\mathrm{T}} = (\mathbf{AB})^{\mathrm{T}} \neq \mathbf{A}^{\mathrm{T}}\mathbf{B}^{\mathrm{T}}$.

4. For the rotation matrix
$$\mathbf{M} = \begin{pmatrix} \cos\theta & -\sin\theta & 0 & 0 \\ \sin\theta & \cos\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix},$$
verify that $\mathbf{M}^{-1} = \mathbf{M}^{\mathrm{T}}$ by showing that $\mathbf{M}^{\mathrm{T}}\mathbf{M} = \mathbf{I}$.

5. For the scaling matrix
$$\mathbf{M}_1 = \begin{pmatrix} s_x & 0 & 0 & 0 \\ 0 & s_y & 0 & 0 \\ 0 & 0 & s_z & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix},$$
verify that the matrix
$$\mathbf{M}_2 = \begin{pmatrix} \frac{1}{s_x} & 0 & 0 & 0 \\ 0 & \frac{1}{s_y} & 0 & 0 \\ 0 & 0 & \frac{1}{s_z} & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix},$$
is the inverse of $\mathbf{M}_1$ by showing that $\mathbf{M}_1\mathbf{M}_2 = \mathbf{I}$.

6. A *reflection* matrix is a scaling matrix in which each scaling factor is ± 1, with an odd number of -1s. If $\mathbf{M}$ is a reflection matrix, what is the inverse transpose of $\mathbf{M}$?

7. Let $\mathbf{M}$ be a 3×3 orthonormal matrix whose rows are the vectors $\mathbf{m}_1, \mathbf{m}_2$, and $\mathbf{m}_3$. Verify that $\mathbf{M}$ maps the vectors $\mathbf{m}_1, \mathbf{m}_2$, and $\mathbf{m}_3$ into the basis vectors $\mathbf{i}, \mathbf{j}$, and $\mathbf{k}$, and that $\mathbf{M}^{\mathrm{T}}$ maps the basis vectors $\mathbf{i}, \mathbf{j}$, and $\mathbf{k}$ into the vectors $\mathbf{m}_1, \mathbf{m}_2$, and $\mathbf{m}_3$, thereby demonstrating that $\mathbf{M}^{\mathrm{T}}$ is the inverse of $\mathbf{M}$.