Pushdown Automata - Examples

Lecture 18
Section 2.2

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Outline

1. Homework Review
2. Examples of PDAs
3. Assignment
Exercise 2.10, page 129.

Give an informal description of a pushdown automaton that recognizes the language

\[ A = \{ a^i b^j c^k \mid i = j \text{ or } j = k \text{ where } i, j, k \geq 0 \}. \]
The language $A$ is the union of the languages

$$\{ a^i b^i c^k \mid i, k \geq 0 \}$$

and

$$\{ a^i b^k c^k \mid i, k \geq 0 \}.$$
Solution

A PDA for the first of these languages is

```
\[ \varepsilon, \varepsilon \rightarrow \$ \]
\[ \varepsilon, \varepsilon \rightarrow \varepsilon \]
\[ a, \varepsilon \rightarrow a \]
\[ b, a \rightarrow \varepsilon \]
\[ c, \varepsilon \rightarrow \varepsilon \]
```

A PDA for the second language is

\[
\begin{align*}
a, \varepsilon & \rightarrow \varepsilon \\
\varepsilon, \varepsilon & \rightarrow \$ \\
b, \varepsilon & \rightarrow b \\
\varepsilon, \varepsilon & \rightarrow \varepsilon \\
c, b & \rightarrow \varepsilon \\
\varepsilon, \$ & \rightarrow \varepsilon
\end{align*}
\]
Therefore, a PDA for the language \( A \) is
Example (Pushdown automaton)

- Design a PDA that accepts the language

\[ \{ w \mid w \text{ contains an equal number of a's and b's} \} \]
The strategy will be to keep the excess symbols, either a’s or b’s, on the stack.

One state will represent an excess of a’s.

Another state will represent an excess of b’s.

We can tell when the excess switches from one symbol to the other because at that point the stack will be empty.

In fact, when the stack is empty, we may return to the start state.
Examples

Example (Pushdown automaton)

- $ \xrightarrow{\epsilon} e$
- $a, e \rightarrow a$
- $b, a \rightarrow \epsilon$
- $a \rightarrow a \Leftarrow b < b$
- $b \rightarrow b \Leftarrow a > b$
- $a, \epsilon \rightarrow \epsilon$
- $b, \epsilon \rightarrow \epsilon$
- $a \rightarrow \epsilon$
- $b \rightarrow \epsilon$
- $a \rightarrow \epsilon$
- $b \rightarrow \epsilon$
- $a \rightarrow \epsilon$
- $b \rightarrow \epsilon$
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- $a \rightarrow \epsilon$
- $b \rightarrow \epsilon$
- $a \rightarrow \epsilon$
- $b \rightarrow \epsilon$
- $a \rightarrow \epsilon$
- $b \rightarrow \epsilon$
Example (Pushdown automaton)

- Note that this solution is inspired by the grammar

\[ S \rightarrow SS \mid aSb \mid bSa \mid \varepsilon \]
Example (Pushdown automata)

- Let $\Sigma = \{a, (, )\}$. Design a PDA whose language is
  $$\{w \mid w \text{ contains balanced parentheses}\}.$$

- Let $\Sigma = \{a, b, c, +, *, (, )\}$. Design a PDA whose language is
  $$\{w \mid w \text{ is a valid algebraic expression}\}.$$
Assignment

- Read Section 2.2, pages 112 - 114.
- Problems 21, 22, 23, 24, page 130.
- Let $\Sigma = \{a, (, ), [, ]\}$. Design a PDA whose language is
  \[\{w \mid w \text{ contains balanced parentheses and brackets}\}\].
- Design a PDA whose language is
  \[\{a^nb^mc^md^n \mid m, n \geq 0\}\].
- Design a PDA whose language is
  \[\{a^nb^m \mid n \neq m\}\].