Examples of PDAs
- Equal Number of a’s and b’s
- Balanced Parentheses
- Algebraic Expressions
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Assignment
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Assignment
Example (Pushdown automaton)

Design a PDA that accepts the language

\[ \{ w \mid w \text{ contains an equal number of } a\text{’s and } b\text{’s} \} \]
The strategy will be to keep the excess symbols, either a’s or b’s, on the stack.

One state will represent an excess of a’s.

Another state will represent an excess of b’s.

We can tell when the excess switches from one symbol to the other because at that point the stack will be empty.

In fact, when the stack is empty, we may return to the start state.
Example (Pushdown automaton)

\[
\begin{align*}
&\text{a, } \epsilon \rightarrow a \\
&\text{b, } a \rightarrow \epsilon \\
&\text{b, } a \rightarrow \epsilon \\
&\text{a, } \epsilon \rightarrow b \\
&\text{b, } \epsilon \rightarrow b \\
&\text{b, } \epsilon \rightarrow b \\
&\text{a, } \epsilon \rightarrow \text{a} \\
&\text{b, } \epsilon \rightarrow \text{b} \\
&\text{b, } \epsilon \rightarrow \text{b} \\
&\text{a, } \epsilon \rightarrow \text{a} \\
&\text{b, } \epsilon \rightarrow \text{b} \\
&\text{a, } \epsilon \rightarrow \text{a} \\
&\text{b, } \epsilon \rightarrow \text{b} \\
&\text{a, } \epsilon \rightarrow \text{a} \\
&\text{b, } \epsilon \rightarrow \text{b} \\
&\text{a, } \epsilon \rightarrow \text{a} \\
&\text{b, } \epsilon \rightarrow \text{b} \\
\end{align*}
\]
Example (Pushdown automaton)

Note that this solution is inspired by the grammar

\[
S \rightarrow SS \mid aSb \mid bSa \mid \varepsilon
\]
1. Examples of PDAs
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2. Assignment
Example (Pushdown automata)

Let $\Sigma = \{a, (, )\}$. Design a PDA whose language is

$\{w \mid w \text{ contains balanced parentheses}\}$. 

1. Examples of PDAs
   - Equal Number of a’s and b’s
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2. Assignment
Example (Pushdown automata)

Let $\Sigma = \{a, b, c, +, \times, (, )\}$. Design a PDA whose language is

$\{w \mid w \text{ is a valid algebraic expression}\}$. 

1 Examples of PDAs
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2 Assignment
Assignment

- Read Section 2.2, pages 112 - 114.
- Problems 21, 22, 23, 24, page 130.
- Let $\Sigma = \{a, (, ), [, ]\}$. Design a PDA whose language is
  \[ \{w \mid w \text{ contains balanced parentheses and brackets}\} \].
- Design a PDA whose language is
  \[ \{a^n b^m c^m d^n \mid m, n \geq 0\} \].
- Design a PDA whose language is
  \[ \{a^n b^m \mid n \neq m\} \].