1. Closure Properties of Decidable Languages
   - Intersection
   - Union

2. Closure Properties of Recognizable Languages
   - Intersection
   - Union

3. Assignment
Outline

1. Closure Properties of Decidable Languages
   - Intersection
   - Union

2. Closure Properties of Recognizable Languages
   - Intersection
   - Union

3. Assignment
The class of decidable languages is closed under:

- Union
- Intersection
- Complementation
- Concatenation
- Star
1. Closure Properties of Decidable Languages
   - Intersection
   - Union

2. Closure Properties of Recognizable Languages
   - Intersection
   - Union

3. Assignment
Closure Under Intersection

**Theorem**

If $L_1$ and $L_2$ are decidable, then $L_1 \cap L_2$ is decidable.
Proof.

- Let $D_1$ be a decider for $L_1$ and let $D_2$ be a decider for $L_2$.
- Then build a decider $D$ for $L_1 \cap L_2$ according to the following diagram.
Outline

1. Closure Properties of Decidable Languages
   - Intersection
   - Union

2. Closure Properties of Recognizable Languages
   - Intersection
   - Union

3. Assignment
Theorem

If $L_1$ and $L_2$ are decidable, then $L_1 \cup L_2$ is decidable.
Proof.

- Let $D_1$ be a decider for $L_1$ and let $D_2$ be a decider for $L_2$.
- Then build a decider $D$ for $L_1 \cup L_2$ according to the following diagram.
Theorem

If $L_1$ and $L_2$ are decidable, then $\overline{L}_1$, $L_1L_2$, and $L_1^*$ are decidable.

How would we prove this theorem?
Outline

1. Closure Properties of Decidable Languages
   - Intersection
   - Union

2. Closure Properties of Recognizable Languages
   - Intersection
   - Union

3. Assignment
The class of recognizable languages is closed under

- Union
- Intersection
- Concatenation
- Star
Outline

1 Closure Properties of Decidable Languages
   • Intersection
   • Union

2 Closure Properties of Recognizable Languages
   • Intersection
   • Union

3 Assignment
Closure Under Intersection

**Theorem**

*If $L_1$ and $L_2$ are recognizable, then $L_1 \cap L_2$ is recognizable.*
Proof.

- Let $R_1$ be a recognizer for $L_1$ and let $R_2$ be a recognizer for $L_2$.
- Then build a recognizer $R$ for $L_1 \cap L_2$ according to the following diagram.
Outline

1 Closure Properties of Decidable Languages
   • Intersection
   • Union

2 Closure Properties of Recognizable Languages
   • Intersection
   • Union

3 Assignment
Theorem

If $L_1$ and $L_2$ are recognizable, then $L_1 \cup L_2$ is recognizable.
Proof.

- Let $R_1$ be a recognizer for $L_1$ and let $R_2$ be a recognizer for $L_2$.
- Then build a recognizer $R$ for $L_1 \cup L_2$ according to the following diagram.
Closure of Union

Proof.

- In that diagram, we must be careful to alternate execution between $R_1$ and $R_2$. 
Theorem

If $L_1$ and $L_2$ are recognizable, then $L_1L_2$ and $L_1^*$ are recognizable.

- How would we prove this theorem?
- Why is $\overline{L_1}$ not necessarily recognizable?
Outline

1. Closure Properties of Decidable Languages
   - Intersection
   - Union

2. Closure Properties of Recognizable Languages
   - Intersection
   - Union

3. Assignment
Assignment

- Read Section 3.2, pages 152 - 154.
- Problems 15, 16, page 161.
- Use the program Universal TM.exe to implement Turing Machines to do the following programs.
- Programs are due Fri, Nov 9.

**tm1:** Decrement the input. That is, evaluate the function

\[ f(n) = \begin{cases} 
  n - 1 & \text{if } n > 0, \\
  0 & \text{if } n = 0. 
\end{cases} \]
Assignment

**tm2:** Determine whether the input is a multiple of 3. That is, evaluate the function

\[
    f(n) = \begin{cases} 
    0 & \text{if } n \mod 3 \neq 0, \\
    1 & \text{if } n \mod 3 = 0. 
\end{cases}
\]

**tm3:** Compare two integers \( n_1 \) and \( n_2 \) and determine whether \( n_1 < n_2, n_1 = n_2, \) or \( n_1 > n_2. \) That is, evaluate the function

\[
    f(n_1, n_2) = \begin{cases} 
    < & \text{if } n_1 < n_2, \\
    = & \text{if } n_1 = n_2, \\
    > & \text{if } n_1 > n_2. 
\end{cases}
\]

The function should read \( ▷n_1 \#n_2 \) and replace it with \( ▷<, ▷=, \) or \( ▷>. \)