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2. The Acceptance Problem for DFAs
3. The Emptiness Problem for DFAs
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6. Assignment
1 Decision Problems
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Definition (Decision Problem)
A decision problem is a question that has a yes-or-no answer.

Definition (Decidable)
A decision problem is decidable if there is an algorithm for it that will produce the correct yes-or-no answer for every instance of the problem.
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The Acceptance Problem for DFAs

Definition (The Acceptance Problem for DFAs)

Given a DFA $M$ and a string $w$, does $M$ accept $w$?
The Acceptance Problem for DFAs

To decide the problem, we let $w$ be the input to $M$ and see whether we end up in an accepting state.
The Acceptance Problem for DFAs

- To decide the problem, we let \( w \) be the input to \( M \) and see whether we end up in an accepting state.
- Therefore, the Acceptance Problem for DFAs is decidable.
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Definition (The Emptiness Problem for DFAs)

Given a DFA $M$, is the language of $M$ empty. That is, does $M$ reject every word in $\Sigma^*$?
The Emptiness Problem for DFAs

- The strategy is to do a breadth-first search of the state diagram for an accept state, starting from the start state.
The Emptiness Problem for DFAs

- If the start state is an accept state, then the answer is “no.”
The Emptiness Problem for DFAs

- If the start state is an accept state, then the answer is “no.”
- If not, then mark the start state as “inspected.”

Continue in this manner with the states that are reachable from the marked states in one transition and that have not yet been marked.
The Emptiness Problem for DFAs

- If the start state is an accept state, then the answer is “no.”
- If not, then mark the start state as “inspected.”
- Then inspect every state that is reachable in one transition from the start state and is not yet marked.
The Emptiness Problem for DFAs

- If the start state is an accept state, then the answer is “no.”
- If not, then mark the start state as “inspected.”
- Then inspect every state that is reachable in one transition from the start state and is not yet marked.
- If any of them is an accept state, then the answer is “no.”
If the start state is an accept state, then the answer is “no.”
If not, then mark the start state as “inspected.”
Then inspect every state that is reachable in one transition from the start state and is not yet marked.
If any is an of them is an accept state, then the answer is “no.”
If not, then mark them as inspected.
The Emptiness Problem for DFAs

- If the start state is an accept state, then the answer is “no.”
- If not, then mark the start state as “inspected.”
- Then inspect every state that is reachable in one transition from the start state and is not yet marked.
- If any of them is an accept state, then the answer is “no.”
- If not, then mark them as inspected.
- Continue in this manner with the states that are reachable from the marked states in one transition and that have not yet been marked.
This procedure will eventually terminate when
- It reaches an accept state, in which case the answer is “no.”
- It can reach only states that are already marked, none of which are accept states, in which case the answer is “yes.”
The Emptiness Problem for DFAs

- This procedure will eventually terminate when
  - It reaches an accept state, in which case the answer is “no.”
  - It can reach only states that are already marked, none of which are accept states, in which case the answer is “yes.”

- Therefore, the Emptiness Problem for DFAs is decidable.
Outline

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Definition (The Equivalence Problem for DFAs)

Given two DFAs $M_1$ and $M_2$, do they have the same language? That is, does $L(M_1) = L(M_2)$?
The Equivalence Problem for DFAs

- The strategy is to follow the algorithm to build the DFA $M$ whose language is

$$\left( L(M_1) \cap \overline{L(M_2)} \right) \cup \left( \overline{L(M_1)} \cap L(M_2) \right).$$

Then solve the Emptiness Problem for $M$.

If $L(M) = \emptyset$, then $L(M_1) = L(M_2)$.

If $L(M) \neq \emptyset$, then $L(M_1) \neq L(M_2)$.

Therefore, the Equivalence Problem for DFAs is decidable.
The Equivalence Problem for DFAs

- The strategy is to follow the algorithm to build the DFA $M$ whose language is
  \[
  (L(M_1) \cap \overline{L(M_2)}) \cup (\overline{L(M_1)} \cap L(M_2)).
  \]
- Then solve the Emptiness Problem for $M$.

Therefore, the Equivalence Problem for DFAs is decidable.
The strategy is to follow the algorithm to build the DFA $M$ whose language is

$$\left( L(M_1) \cap \overline{L(M_2)} \right) \cup \left( \overline{L(M_1)} \cap L(M_2) \right).$$

Then solve the Emptiness Problem for $M$.

- If $L(M) = \emptyset$, then $L(M_1) = L(M_2)$.
The Equivalence Problem for DFAs

- The strategy is to follow the algorithm to build the DFA $M$ whose language is

$$\left(L(M_1) \cap \overline{L(M_2)}\right) \cup \left(\overline{L(M_1)} \cap L(M_2)\right).$$

- Then solve the Emptiness Problem for $M$.
  - If $L(M) = \emptyset$, then $L(M_1) = L(M_2)$.
  - If $L(M) \neq \emptyset$, then $L(M_1) \neq L(M_2)$.

Therefore, the Equivalence Problem for DFAs is decidable.
The Equivalence Problem for DFAs

- The strategy is to follow the algorithm to build the DFA $M$ whose language is

$$\left( L(M_1) \cap \overline{L(M_2)} \right) \cup \left( \overline{L(M_1)} \cap L(M_2) \right).$$

- Then solve the Emptiness Problem for $M$.
  - If $L(M) = \emptyset$, then $L(M_1) = L(M_2)$.
  - If $L(M) \neq \emptyset$, then $L(M_1) \neq L(M_2)$.

- Therefore, the Equivalence Problem for DFAs is decidable.
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Other decision problems for DFAs:

- Decide whether \( L(M) = \Sigma^* \).
- Decide whether \( M \) accepts any string of even length.
- Decide whether \( L(M) \) is infinite.
- Decide whether \( L = L^* \).
- Decide whether \( L = L^R \).
- Decide whether \( L(M_1) \subseteq L(M_2) \).
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Assignment

- Section 4.2 Exercises 1, 2, 4, 6, 9, 14, 17, 18.