LR Parsing - The Tables

Lecture 11

Sections 4.5, 4.7

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Wed, Feb 18, 2009

Outline

The LR(0) Parsing Tables

- There are two tables that we will construct.
- The action table contains shift and reduce actions to be taken upon processing terminals.
- The goto table contains changes of state upon matching productions.

The Action Table

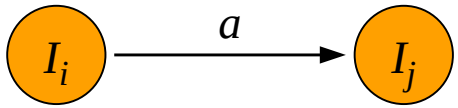
- The action table contains one row for each state in the PDA and one column for each terminal and EOF ($).
- The entries are
  - Shift n.
    - Push the current token and move to state n.
  - Reduce n.
    - Pop the symbols of the right side of production n and then push the nonterminal of the left side.
    - Then change state according to the goto table.

Building the Action Table

- The action table is built from items of the form $[A \to \alpha \bullet a\beta]$ and $[A \to \alpha \bullet]$.
  - Items $[A \to \alpha \bullet a\beta]$ produce shift operations.
  - Items $[A \to \alpha \bullet]$ produce reduce operations.
- The goto table is built from items of the form $[A \to \alpha \bullet B\beta]$.

Building the Action Table

- If $[A \to \alpha \bullet a\beta]$ is in state I_i and the PDA transition on a is from state I_i to state I_j, then set the (i, a) entry to "shift j."
- In the table, we write "sj," where j is the number of the destination state.
- Thus, every transition on a terminal becomes a shift entry in the action table.



Building the Action Table

- If $[A \to \alpha \bullet]$ is in state I_i and $A \neq S'$, then set the (i, a) entry to "reduce $A \to \alpha$" for all a in FOLLOW(A).
- In the table, we write "rk," where k is the number of the production $A \to \alpha$.

Building the Action Table

- If $[S' \to S \bullet]$ is in state I_i, then the $(i, \$)$ entry is "accept."
- In the table, we write "acc."
- All empty cells are labeled "error."

Shift/Reduce Conflicts

- It is possible that a cell will contain both a shift operation and a reduce operation.
- This is called a shift/reduce conflict.
- To choose between "shift" and "reduce," each case must be considered on its own merit.
- Consider the case of $E \to E + E \mid E * E$ and the inputs $\mathbf{a}+\mathbf{b}*\mathbf{c}$ and $\mathbf{a}*\mathbf{b}+\mathbf{c}$.

Reduce/Reduce Conflicts

- It is possible that a cell will contain two different reduce operations.
- This is called a reduce/reduce conflict.
- This occurs when a sequence of tokens matches the right-hand sides of two different productions at the same time.
- For each such conflict in the table, we must choose which reduction to apply.

Example

Example (FIRST and FOLLOW)

- To find the action table for our example, we need to know nullability, FIRST, and FOLLOW.

$$
\begin{aligned}
E' &\rightarrow E \\
E &\rightarrow E + T \mid T \\
T &\rightarrow T * F \mid F \\
F &\rightarrow (E) \mid \mathbf{id} \mid \mathbf{num}
\end{aligned}
$$

Example

Example (FIRST and FOLLOW)

	Nullable	FIRST	FOLLOW
E'	No	$\{(, \mathbf{id}, \mathbf{num}\}$	$\{\$\}$
E	No	$\{(, \mathbf{id}, \mathbf{num}\}$	$\{\$, +,)\}$
T	No	$\{(, \mathbf{id}, \mathbf{num}\}$	$\{\$, +,), *\}$
F	No	$\{(, \mathbf{id}, \mathbf{num}\}$	$\{\$, +,), *\}$

Example

Example (The Action Table)

- Then number the productions, starting with 0 for the special production:
  0. $E' \to E$
  1. $E \to E + T$
  2. $E \to T$
  3. $T \to T * F$
  4. $T \to F$
  5. $F \to (E)$
  6. $F \to \mathbf{id}$
  7. $F \to \mathbf{num}$

Example

Example (The Action Table)

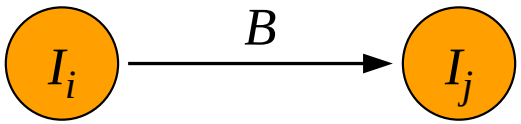
	+	*	(	)	id	num	$
0			s4		s5	s6	
1	s7						acc
2	r2	s8		r2			r2
3	r4	r4		r4			r4
4			s4		s5	s6	
5	r6	r6		r6			r6
6	r7	r7		r7			r7
7			s4		s5	s6	
8			s4	s12	s5	s6	
9	s7			r1			
10	r1	s8		r3			r1
11	r3	r3		r5			r3
12	r5	r5					r5

The Goto Table

- The goto table has one row for each state in the PDA and one column for each nonterminal except S'.
- The entries are states of the PDA.

Building the Goto Table

- If $[A \to \alpha \bullet B\beta]$ is in state I_i and the PDA transition is



  then set the (i, B) entry to "goto j."
- In the goto table, we write "j."
- Thus, every transition on a nonterminal becomes an entry in the goto table.

Example

Example (The Goto Table)

	E	T	F
0	1	2	3
1			
2			
3			
4	9	2	3
5			
6			
7		10	3
8			11
9			
10			
11			
12			

Example

Example (Parse $(\mathbf{id} + \mathbf{num}) * \mathbf{id}$)

Stack	Input
0	$($ **id** $+$ **num** $)$ $*$ **id** \$
0 $($ 4	**id** $+$ **num** $)$ $*$ **id** \$
0 $($ 4 **id** 5	$+$ **num** $)$ $*$ **id** \$
0 $($ 4 F 3	$+$ **num** $)$ $*$ **id** \$
0 $($ 4 T 2	$+$ **num** $)$ $*$ **id** \$
0 $($ 4 E 9	$+$ **num** $)$ $*$ **id** \$
0 $($ 4 E 9 $+$ 7	**num** $)$ $*$ **id** \$
0 $($ 4 E 9 $+$ 7 **num** 6	$)$ $*$ **id** \$
0 $($ 4 E 9 $+$ 7 F 3	$)$ $*$ **id** \$

Example

Example (LR Parsing)

Stack	Input
0 (4 E 9 + 7 T 10	) $*$ **id** \$
0 (4 E 9	) $*$ **id** \$
0 (4 E 9) 12	$*$ **id** \$
0 F 3	$*$ **id** \$
0 T 2	$*$ **id** \$
0 T 2 $*$ 8	**id** \$
0 T 2 $*$ 8 **id** 5	\$
0 T 2 $*$ 8 F 11	\$
0 T 2	\$
0 E 1	\$
Accept	\$

Example

Example (A Simplified Grammar)

- We may simplify our grammar to

$$\begin{aligned} E &\rightarrow E + E \\ E &\rightarrow E * E \\ E &\rightarrow (E) \\ E &\rightarrow \mathbf{id} \\ E &\rightarrow \mathbf{num} \end{aligned}$$

- In this form, the precedence rules for $+$ and $*$ are not implicit.
- They must be incorporated into the tables.

Example

Example (The Action and Goto Tables)

Action								Goto
	$+$	$*$	$($	$)$	$\mathbf{id}$	$\mathbf{num}$	$\$$	E
0			s2		s3	s4		1
1	s5	s6					acc	
2			s2		s3	s4		7
3	r4	r4		r4			r4	
4	r5	r5		r5			r5	
5			s2		s3	s4		8
6			s2		s3	s4		9
7	s5	s6		s10				
8	s5/r1	s6/r1		r1			r1	
9	s5/r2	s6/r2		r2			r2	
10	r3	r3		r3			r3	

Example

Example (Shift/Reduce Conflicts and Associativity)

- The shift/reduce conflict in cell $(8, +)$ is between shifting a $+$ and reducing by

$$E \to E + E.$$

- If we choose "shift," then we will make addition right associative.
- If we choose "reduce," then we will make addition left associative.
- The case is similar in cell $(9, *)$ regarding multiplication.

Example

Example (Shift/Reduce Conflicts and Precedence)

- The shift/reduce conflict in cell $(9, +)$ is between shifting a $+$ and reducing by

$$E \to E * E.$$

- If we choose "shift," then we will give multiplication a higher precedence than addition.
- If we choose "reduce," then we will give addition a higher precedence than multiplication.
- The case is similar in cell $(8, *)$.

Example

Example (The Action and Goto Tables)

Action								Goto
	$+$	$\star$	(	)	**id**	**num**	$	E
0			s2		s3	s4		1
1	s5	s6					acc	
2			s2		s3	s4		7
3	r4	r4		r4			r4	
4	r5	r5		r5			r5	
5			s2		s3	s4		8
6			s2		s3	s4		9
7	s5	s6		s10				
8	r1	s6		r1			r1	
9	r2	r2		r2			r2	
10	r3	r3		r3			r3	

Assignment

Homework

- The grammar

$$R \rightarrow R \cup R \mid RR \mid R* \mid (R) \mid \mathbf{a} \mid \mathbf{b}$$

  generates all regular expressions on the alphabet $\Sigma = \{\mathbf{a}, \mathbf{b}\}$.
- Build the action and goto tables.
- Resolve any conflicts in the table.
- Parse the expression $\mathbf{ab}*(\mathbf{a} \cup \mathbf{b})$.