Apportionment Problems

Lecture 16
Section 4.1

Robb T. Koether

Hampden-Sydney College

Fri, Oct 4, 2013

Outline

Apportionment Problems

Example

- I have a class of 10 students and I have 50 pieces of candy to hand out.

Apportionment Problems

Example

- I have a class of 10 students and I have 50 pieces of candy to hand out.
- I will distribute the candies according to the number of correct answers they give on the next test, out of 12 questions.

Apportionment Problems

Example

- I have a class of 10 students and I have 50 pieces of candy to hand out.
- I will distribute the candies according to the number of correct answers they give on the next test, out of 12 questions.
- The numbers of correct answers are

$$5, 5, 5, 7, 7, 8, 10, 11, 11, 11,$$

for a total of 80 correct answers.

Apportionment Problems

Example

- I have a class of 10 students and I have 50 pieces of candy to hand out.
- I will distribute the candies according to the number of correct answers they give on the next test, out of 12 questions.
- The numbers of correct answers are

$$5, 5, 5, 7, 7, 8, 10, 11, 11, 11,$$

  for a total of 80 correct answers.
- How many candies should each student get?

Apportionment Problems

Example

- I have a class of 10 students and I have 50 pieces of candy to hand out.
- I will distribute the candies according to the number of correct answers they give on the next test, out of 12 questions.
- The numbers of correct answers are

$$5, 5, 5, 7, 7, 8, 10, 11, 11, 11,$$

  for a total of 80 correct answers.
- How many candies should each student get?
- The ratio of 50 : 80 indicates that each correct answer is worth 0.625 candies.

Apportionment Problems

Definition (Apportionment Problem)

The classic apportionment problem involves a representative body where each state is given a certain number of seats, according to the state's population.

- Let N be the number of states.
- Let M the number of seats.
- Let $p_1, p_2, p_3, \ldots, p_N$ be the states' populations.
- Let $P = p_1 + p_2 + p_3 + \cdots + p_N$, the total population.

Definitions

Definition (Standard Divisor)

The standard devisor (SD) is $\frac{P}{M}$. It represents the number of people that each seat represents.

Definition (Standard Quota)

The standard quota of a state is the exact fractional number of seats it should get for its "fair share." It is computed as

$$q_i = \left(\frac{p_i}{P}\right) M = \frac{p_i}{\text{SD}}.$$

Definition (Lower and Upper Quotas)

The lower quota and the upper quota are the two whole numbers nearest the standard quota.

Lower and Upper Quotas

Example (Lower and Upper Quotas)

Find the standard divisor, standard quota, and the lower and upper quotas for the 10 students, with scores

$$5, 5, 5, 7, 7, 8, 10, 11, 11, 11,$$

and the 50 pieces of candy.

Outline

The Round-Off Method

Definition (The Round-Off Method)

1. Calculate each state's standard quota.

The Round-Off Method

Definition (The Round-Off Method)

1. Calculate each state's standard quota.
2. Round each one *down* to the nearest integer to get that state's quota.

The Round-Off Method

Definition (The Round-Off Method)

1. Calculate each state's standard quota.
2. Round each one *down* to the nearest integer to get that state's quota.
3. Cross your fingers that it all works out ok.

Example

Example

- Apply round-off method to the 10 students and the 50 candies.
- The standard quotas are

$$3.125, 3.125, 3.125, 4.375, 4.375, 5.0, 6.25, 6.875, 6.875, 6.875.$$

- What is the final apportionment?
- Did it work out ok?

Outline

Hamilton's Method

Definition (Hamilton's Method)

1. Calculate each state's standard quota.

- Historical background.

Hamilton's Method

Definition (Hamilton's Method)

1. Calculate each state's standard quota.
2. Round each one *down* to the lower quota.

- Historical background.

Hamilton's Method

Definition (Hamilton's Method)

1. Calculate each state's standard quota.
2. Round each one *down* to the lower quota.
3. Distribute the surplus to the states with the largest fraction parts, i.e., the largest difference between their standard quota and their lower quota.

- Historical background.

Example

Example

- Apply Hamilton's method to the 10 students and the 50 candies.
- The standard quotas are

$$3.125, 3.125, 3.125, 4.375, 4.375, 5.0, 6.25, 6.875, 6.875, 6.875.$$

- What is the final apportionment?

Outline

Assignment

Assignment

- Page 128: Exercises 4, 6, 7.
- Page 130: Exercises 18, 19.