

# Annuities

## Lecture 27

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- 1 Assignment
- 2 Annuities
- 3 Building up an Annuity
- 4 Drawing down an Annuity

# Outline

- 1 Assignment
- 2 Annuities
- 3 Building up an Annuity
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# Assignment

## Collected

To be collected on Wednesday, November 5.

- Chapter 10: 52, 56.
- Inflation handout: #4, #6

## Assignment

- See handout.

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# Definitions

## Definition (Annuity (Stolen from Investopedia))

An **annuity** is a financial product sold by financial institutions that is designed to accept and grow funds from an individual and then, upon annuitization, pay out a stream of payments to the individual at a later point in time.

- Typically, a retirement plan is an annuity – You invest over your working life and then withdraw from it during retirement.
- One could establish an annuity to pay for a child's college education – You invest for 18 years and withdraw over the following 4 years.

# Definitions

See a dramatization of an annuity.

- An annuity has two stages.
  - The investment stage.
  - The withdrawal stage.
- During the investment stage, the balance grows.
- During the withdrawal stage, the balance diminishes.



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# First Annuity Formula

- The formula for the future value while investing:

$$F = \frac{P((1 + r)^n - 1)}{r},$$

where  $F$  is the future value of the annuity,  $P$  is the amount investment per period,  $r$  is the interest rate per period, and  $n$  is the number of deposits (one per period).

# Example

## Example (Three Deposits)

- Let the annual interest rate be 10%.
- Invest \$1000 each year for 3 years.

# Example

## Example (Three Deposits)

The future value is

$$F = \frac{P((1+r)^n - 1)}{r}$$

# Example

## Example (Three Deposits)

The future value is

$$\begin{aligned} F &= \frac{P((1+r)^n - 1)}{r} \\ &= \frac{1000((1.10)^3 - 1)}{.10} \end{aligned}$$

# Example

## Example (Three Deposits)

The future value is

$$\begin{aligned} F &= \frac{P((1+r)^n - 1)}{r} \\ &= \frac{1000((1.10)^3 - 1)}{.10} \\ &= 3310.00. \end{aligned}$$

# Example

## Example (Three Deposits)

The investment stage:

Starting Balance	Interest	Total	Investment	Ending Balance
\$0	\$0	\$0	\$1000	\$1000

# Example

## Example (Three Deposits)

The investment stage:

Starting Balance	Interest	Total	Investment	Ending Balance
\$0	\$0	\$0	\$1000	\$1000
\$1000	\$100	\$1100	\$1000	\$2100



# Example

## Example (Three Deposits)

The investment stage:

Starting Balance	Interest	Total	Investment	Ending Balance
\$0	\$0	\$0	\$1000	\$1000
\$1000	\$100	\$1100	\$1000	\$2100
\$2100	\$210	\$2310	\$1000	\$3310

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## Second Annuity Formula

- The formula for the amount to withdraw each period:

$$M = \frac{Ar}{1 - (1 + r)^{-n}},$$

where  $M$  is the amount withdrawn per period,  $A$  is the amount in the annuity when the withdrawals begin,  $r$  is the rate per period, and  $n$  is the number of withdrawals (one per period).

# Example

## Example (Three Withdrawals)

- Continuing the example, the person has accumulated \$3310 after 3 years.
- How much can he withdraw each year for 3 years?

# Example

## Example (Three Withdrawals)

The amount withdrawn is

$$M = \frac{Ar}{1 - (1 + r)^{-n}}$$

# Example

## Example (Three Withdrawals)

The amount withdrawn is

$$\begin{aligned}M &= \frac{Ar}{1 - (1 + r)^{-n}} \\ &= \frac{(3310)(.10)}{1 - (1.10)^3}\end{aligned}$$

# Example

## Example (Three Withdrawals)

The amount withdrawn is

$$\begin{aligned}M &= \frac{Ar}{1 - (1 + r)^{-n}} \\ &= \frac{(3310)(.10)}{1 - (1.10)^3} \\ &= 1331.00.\end{aligned}$$

# Example

## Example (Three Withdrawals)

The withdrawal stage:

Starting Balance	Interest	Total	Withdrawal	Ending Balance
\$3310	\$331	\$3641	\$1331	\$2310



# Example

## Example (Three Withdrawals)

The withdrawal stage:

Starting Balance	Interest	Total	Withdrawal	Ending Balance
\$3310	\$331	\$3641	\$1331	\$2310
\$2310	\$231	\$2541	\$1331	\$1210

# Example

## Example (Three Withdrawals)

The withdrawal stage:

Starting Balance	Interest	Total	Withdrawal	Ending Balance
\$3310	\$331	\$3641	\$1331	\$2310
\$2310	\$231	\$2541	\$1331	\$1210
\$1210	\$121	\$1331	\$1331	\$0

# Another Example

## Example

- A person earning \$48,000 a year invests 5% of his income in a retirement account earning 9% per year for 45 years.
- How much does he have at the end of 45 years?
- He retires and expects to live for 20 more years.
- How much can he withdraw each year for 20 years?

# Another Example

## Example

- A person earning \$48,000 a year invests 5% of his income in a retirement account earning 9% per year for 45 years.
- How much does he have at the end of 45 years?
- He retires and expects to live for 20 more years.
- How much can he withdraw each year for 20 years?
- What if the interest rate had been 15%?

# Another Example

## Example

- A person earning \$48,000 a year invests 5% of his income in a retirement account earning 9% per year for 45 years.
- How much does he have at the end of 45 years?
- He retires and expects to live for 20 more years.
- How much can he withdraw each year for 20 years?
- What if the interest rate had been 15%?
- What if the inflation rate were 3%?