

The Pairwise-Comparison Method

Lecture 10
Section 1.5

Robb T. Koether

Hampden-Sydney College

Mon, Sep 11, 2017

- 1 The Method of Pairwise Comparisons
- 2 Examples
- 3 The Number of Comparisons
- 4 A Shortcoming of the Method
- 5 Assignment

Outline

- 1 The Method of Pairwise Comparisons
- 2 Examples
- 3 The Number of Comparisons
- 4 A Shortcoming of the Method
- 5 Assignment

The Method of Pairwise Comparisons

Definition (The Method of Pairwise Comparisons)

By the **method of pairwise comparisons**, each voter ranks the candidates. Then, **for every pair** (for every possible two-way race) of candidates,

- Determine which one was preferred more often.
- That candidate gets 1 point.
- If there is a tie, each candidate gets $1/2$ point.

The candidate who gets the greatest number of points is the winner.

Then rank the candidates according to the number of points received.

Outline

1 The Method of Pairwise Comparisons

2 Examples

3 The Number of Comparisons

4 A Shortcoming of the Method

5 Assignment

Example

Example

Suppose that there are 3 candidates: A, B, C. The following table summarizes the voters' preferences.

No. of voters	Preferences			
	7	6	3	2
1st	A	B	B	C
2nd	B	A	C	B
3rd	C	C	A	A

- How many pairings are there?
- List the pairings.
- Count the votes for each pairing and determine the winner.

Example

Example

Suppose that there are 4 candidates: A, B, C, D. The following table summarizes the voters' preferences.

No. of voters	Preferences			
	11	8	7	4
1st	A	B	D	C
2nd	B	D	A	A
3rd	C	C	B	D
4th	D	A	C	B

- How many pairings are there?
- List the pairings.
- Count the votes for each pairing and determine the winner.

Example

Example

Suppose that there are 5 candidates: A, B, C, D, E. The following table summarizes the voters' preferences.

No. of voters	Preferences						
	6	4	4	4	2	1	1
1st	B	B	D	E	A	C	C
2nd	A	A	A	C	D	B	D
3rd	C	D	E	D	C	A	A
4th	D	E	C	B	B	D	B
5th	E	C	B	A	E	E	E

- How many pairings are there?
- List the pairings.
- Count the votes for each pairing and determine the winner.

Outline

- 1 The Method of Pairwise Comparisons
- 2 Examples
- 3 The Number of Comparisons**
- 4 A Shortcoming of the Method
- 5 Assignment

The Number of Comparisons

- How many comparisons are there?
 - With 3 candidates, there are 3 comparisons.
 - With 5 candidates, there are 10 comparisons.

The Number of Comparisons

- How many comparisons are there?
 - With 3 candidates, there are 3 comparisons.
 - With 5 candidates, there are 10 comparisons.
 - With 6 candidates, how many comparisons would there be?

The Number of Comparisons

- How many comparisons are there?
 - With 3 candidates, there are 3 comparisons.
 - With 5 candidates, there are 10 comparisons.
 - With 6 candidates, how many comparisons would there be?
 - How many with 7 candidates?

Outline

- 1 The Method of Pairwise Comparisons
- 2 Examples
- 3 The Number of Comparisons
- 4 A Shortcoming of the Method**
- 5 Assignment

A Shortcoming

- This method seems to take pretty much everything into account.
- So what could go wrong?

A Shortcoming

Example (A Shortcoming)

No. of voters	Preferences						
	6	4	4	4	2	1	1
1st	B	B	D	E	A	C	C
2nd	A	A	A	C	D	B	D
3rd	C	D	E	D	C	A	A
4th	D	E	C	B	B	D	B
5th	E	C	B	A	E	E	E

- Reconsider the previous example.

A Shortcoming

Example (A Shortcoming)

No. of voters	Preferences						
	6	4	4	4	2	1	1
1st	B	B	D	E	A	C	C
2nd	A	A	A	C	D	B	D
3rd	C	D	E	D	C	A	A
4th	D	E	C	B	B	D	B
5th	E	C	B	A	E	E	E

- At the last minute, candidate C drops out.

A Shortcoming

Example (A Shortcoming)

No. of voters	Preferences						
	6	4	4	4	2	1	1
1st	B	B	D	E	A	B	D
2nd	A	A	A	D	D	A	A
3rd	D	D	E	B	B	D	B
4th	E	E	B	A	E	E	E

- Now who is the winner?

A Shortcoming

Example (A Shortcoming)

No. of voters	Preferences						
	6	4	4	4	2	1	1
1st	B	B	D	E	A	B	D
2nd	A	A	A	D	D	A	A
3rd	D	D	E	B	B	D	B
4th	E	E	B	A	E	E	E

- Now who is the winner?
- Is that surprising?

The Perfect Voting Method

- Is there a voting method that has no shortcoming?

Outline

- 1 The Method of Pairwise Comparisons
- 2 Examples
- 3 The Number of Comparisons
- 4 A Shortcoming of the Method
- 5 Assignment**

Assignment

Assignment

- Chapter 1: Exercises 41, 42, 43, 44, 45, 47, 49, 50.