Street-Routing Problems
Lecture 29
Sections 5.2

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1. Street-Routing Problems
   - The Security Guard Problem
   - The Mail Carrier Problem
   - The Königsberg Problem
   - The Bridges of Madison County Problem

2. Examples

3. Euler Paths and Circuits

4. Definitions

5. Euler’s Theorems

6. Assignment
We have already considered the Traveling Salesman Problem. There are many other problems that would fall under the heading “Street-Routing Problems.” We will consider four of them.
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We will consider four of them.

- The Security Guard Problem
- The Mail Carrier Problem
- The Königsberg Bridge Problem
- The Bridges of Madison County
Outline

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A security guard must patrol every street of the neighborhood.

What route should he follow to minimize the total distance?

Must he walk some streets twice?
Street-Routing Problems

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Examples

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Assignment
The Mail Carrier Problem

Example (The Mail Carrier Problem)

A mail carrier must deliver mail to *both sides* of every street of the neighborhood, except the boundary.

What route should he follow to minimize the total distance?

Must he walk some streets twice?
Street-Routing Problems

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Assignment
A Königsberger would like to take a stroll across the seven bridges.
Can it be done without ever crossing the same bridge twice?
Does it matter where the stroller starts?
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6 Assignment
A photographer wants to photograph each of the 11 bridges of Madison County.

He must cross the bridge to photograph it and each bridge has a $5 toll.

What route will minimize the total cost?
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He must cross the bridge to photograph it and each bridge has a $5 toll.

What route will minimize the total cost?
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In the Security Guard Problem, we want a path that traverses every edge at least once and has the minimal total length.
In the Mail Carrier Problem, we want a path that traverses every edge \textit{at least twice} (except the boundary) and has the minimal total length.

With the duplicated edges, the Mail Carrier Problem is the same as the Security Guard Problem.
In the Bridges of Königsberg Problem, we want a circuit that traverses each edge *exactly once*. 
We draw a graph that shows only the relevant parts.
Example (The Bridges of Madison County Problem)

- In the Bridges of Madison County Problem, we want a circuit that traverses each edge \textit{at least once} and has the minimal total length.
- This is the same as the Security Guard Problem.
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6 Assignment
Definition (Euler Path)
An Euler path (pronounced "oiler") is a path that traverses each edge exactly once.

Definition (Euler Circuit)
An Euler circuit is an Euler path that is a circuit.
In the Bridges of Königsberg Problem, we seek an Euler path and an Euler circuit.
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Definitions

Definition (Connected)
A graph is connected if, for any two vertices, there is a path from one to the other.

Definition (Even and Odd Vertices)
A vertex is even if an even number of edges emanate from it. Otherwise, it is odd.
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6 Assignment
Euler’s Theorems

Theorem (Euler Circuits)
If a graph is connected and every vertex is even, then it has an Euler circuit. Otherwise, it does not have an Euler circuit.

Theorem (Euler Paths)
If a graph is connected and it has exactly 2 odd vertices, then it has an Euler path. If it has more than 2 odd vertices, then it does not have an Euler path.
Connected and Disconnected
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Chapter 5: Exercises 19, 20, 21, 22, 27.