

Sampling Distribution of a Sample Proportion

Lecture 26
Section 8.4

Robb T. Koether

Hampden-Sydney College

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Outline

- 1 The Central Limit Theorem for Proportions
- 2 Applications
- 3 Assignment

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1 The Central Limit Theorem for Proportions

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3 Assignment

The Central Limit Theorem for Proportions

Theorem (The Central Limit Theorem for Proportions)

- For any population, the sampling distribution of \hat{p} has the following mean and standard deviation:

$$\begin{aligned}\mu_{\hat{p}} &= p \\ \sigma_{\hat{p}} &= \sqrt{\frac{p(1-p)}{n}}.\end{aligned}$$

- Furthermore, the sampling distribution of \hat{p} is approximately normal, provided n is large enough.

The Central Limit Theorem for Proportions

The Sample Size

- n is large enough if

$$np \geq 5 \text{ and } n(1 - p) \geq 5.$$

Outline

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Applications

- Suppose that 60% of all high-school students own a cell phone.
- If we survey 150 high-school students, how likely is it that we will find that at least 65% of them own a cell phone?

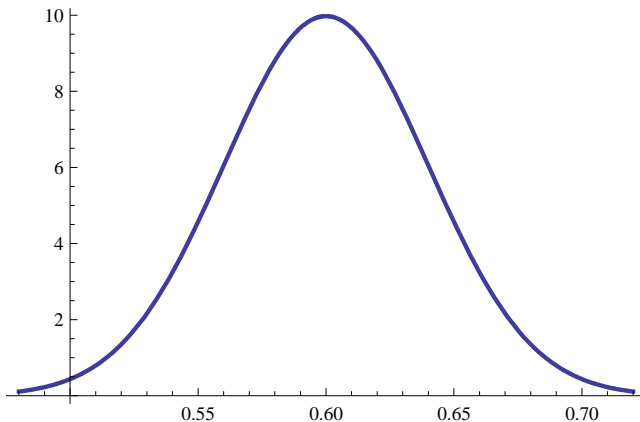
Applications

- If $p = 0.60$ and our sample size is $n = 150$, then \hat{p} is normal with mean $\mu_{\hat{p}} = 0.60$ and

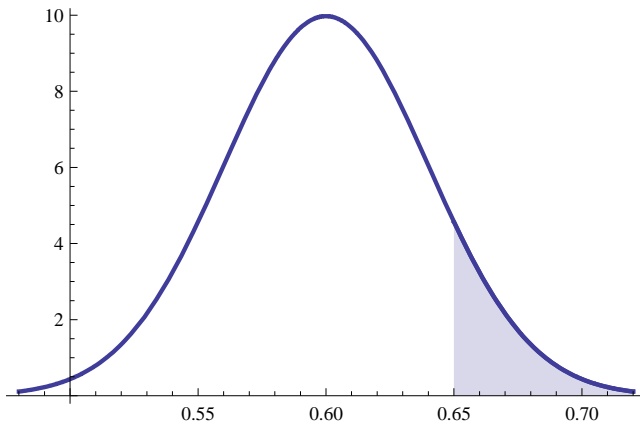
$$\sigma_{\hat{p}} = \sqrt{\frac{(0.60)(0.40)}{150}} = \sqrt{0.0016} = 0.04.$$

- Want to know the probability that $\hat{p} \geq 0.65$.

Applications



Applications



Applications

- The probability that \hat{p} is greater than 0.65 is

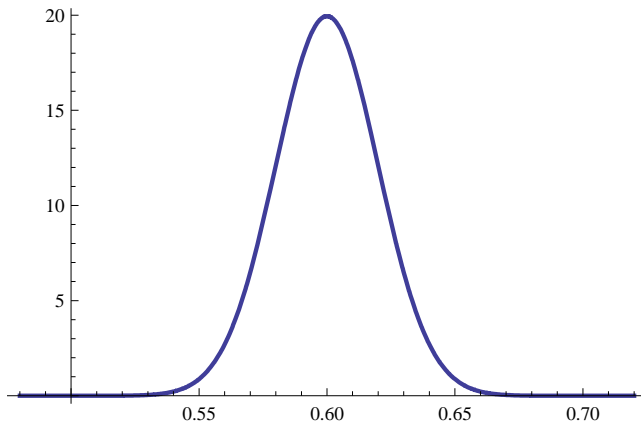
$$\text{normalcdf}(.65, E99, .60, .04) = 0.1056.$$

Applications

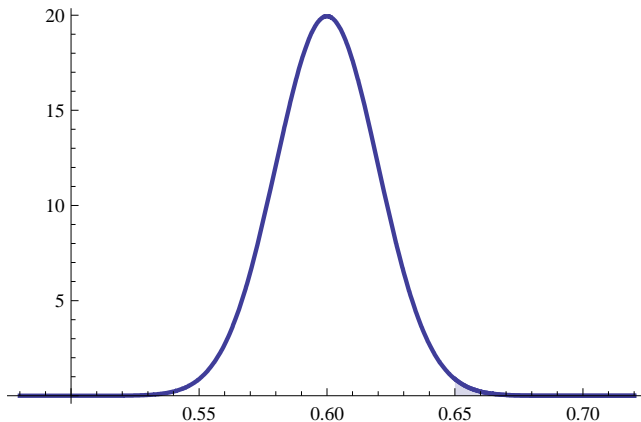
- What if our sample size were 600 instead of 150?
- Then \hat{p} is normal with mean $\mu_{\hat{p}} = 0.60$ and

$$\sigma_{\hat{p}} = \sqrt{\frac{(0.60)(0.40)}{600}} = \sqrt{0.0004} = 0.02.$$

Applications



Applications



Applications

- The probability that \hat{p} is greater than 0.65 is

$$\text{normalcdf}(.65, E99, .60, .02) = 0.0062.$$

Applications

- Suppose that the true proportion of Americans who approve of President Obama's performance is 45%.
- Can we use a sample of 1500 Americans to disprove the hypothesis that the rate is 40%?

Applications

- Which is the correct question to ask?
 - Given that we observed $\hat{p} = 0.45$ in a sample, what is the probability that $p = 0.40$?
 - If p really is 0.40, what is the probability that we would observe $\hat{p} = 0.45$ (or more extreme)?

Applications

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 - Given that we observed $\hat{p} = 0.45$ in a sample, what is the probability that $p = 0.40$?
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Hypothesis Testing

- Let us test the hypotheses

H_0 : 40% of all Americans approve of President Obama's performance.

H_1 : More than 40% of all Americans approve of President Obama's performance.

Hypothesis Testing

- Using the Central Limit Theorem, the null hypothesis predicts that the distribution of \hat{p} is
 - Normal, with
 - Mean p , which is 0.40 (hypothetically).
 - Standard deviation $\sqrt{\frac{(0.40)(0.60)}{1500}} = 0.01265$.
- That is, \hat{p} is $N(0.40, 0.01265)$.
- Find the probability that $\hat{p} \geq 0.45$.
- Design a decision rule so that $\alpha = 0.05$.

Hypothesis Testing

- The critical value will be the 95th percentile of \hat{p} .
- Critical value = `invNorm(.95, .40, .01265)` = 0.4208.
- Therefore, a value of 0.45 would be statistically significant.

Hypothesis Testing

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Hypothesis Testing

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- Each question has 5 choices.
- A student gets a score of 6.
- Can we show statistically that he did not guess at the answers?

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Assignment

Homework

- Read Sections 8.1 - 8.2, pages 499 - 508.
- Exercises 7 - 14, page 526.

Assignment

Homework Answers

8. (a) 0.0264.
(b) Yes. The probability of that is 3.117×10^{-6} .
10. (a) The sampling distribution for the sample proportion of men is normal with mean 0.49 and standard deviation 0.04999.
(b) 0.0548.

Assignment

Homework Answers

12. (a) Draw a normal curve with mean 0.60 and standard deviation 0.04899.
- (b) (i) 0.55101 and 0.64899.
(ii) 0.50202 and 0.69798.
(iii) 0.45303 and 0.74697.
- (c) No. According to part (b)(ii), a proportion as low as 0.50 has only about a $2\frac{1}{2}\%$ chance of occurring.
- (d) Sketch a normal curve with mean 0.60 and standard deviation 0.02449.
- (i) It is narrower.
(ii) They will each be half as wide as before.

Assignment

Homework Answers

14. (a) It is normal with mean 0.50 and standard deviation 0.02887.
(b) $\hat{p} = 0.56$. p -value of 0.56 is 0.0188.
(c) Reject H_0 . A majority of shoppers favor longer shopping hours.