Student's t Distribution

Lecture 34
Section 10.2

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Outline

Introduction

- First we will see how to test a hypothesis concerning the mean on the TI-83 when σ is known.
- However, it is not realistic to assume that we know the value of σ.
- Typically, we use s as an approximation to σ in the formula
$$\frac{\overline{x} - \mu}{\sigma / \sqrt{n}}.$$
- For large samples, we can expect s to be very close to σ, so the substitution will not make any difference.
- However, for smaller samples, the difference will be noticeable.

Hypothesis Testing on the TI-83

- Press `STAT`.
- Select `TESTS`.
- Select `Z-Test`. Press `ENTER`. A window appears requesting information.
- Select `Data` if you have the sample data entered into a list.
- Otherwise, select `Stats`.

Hypothesis Testing on the TI-83
The `Stats` Option

- Assuming you selected `Stats`,
- Enter μ_0, the hypothetical mean.
- Enter σ. (Remember, σ is known.)
- Enter $\overline{x}$.
- Enter n, the sample size.
- Select the type of alternative hypothesis.
- Select `Calculate` and press `ENTER`.

- A window appears with the following information.
  - The title Z-Test.
  - The alternative hypothesis.
  - The value of the test statistic Z.
  - The p-value of the test.
  - The sample mean.
  - The sample size.

Example
The `Stats` Option

- Re-do Example 10.1 on the TI-83 using `Stats`.
- The TI-83 reports that
  - $z = -2.575$.
  - p-value $= 0.005012$.

Hypothesis Testing on the TI-83
The `Data` Option

- Suppose we had selected `Data` instead of `Stats`.
- Then somewhat different information is requested.
- Enter the hypothetical mean.
- Enter σ. (Why?)
- Identify the list that contains the data.
- Skip `Freq` (it should be `1`).
- Select the alternative hypothesis.
- Select `Calculate` and press `ENTER`.

Example
The Data Option

- Re-do Example 10.1 on the TI-83 using Data.
- Enter the data in the chart on page 616 into list L_1.
  - The TI-83 reports that
  - $z = -2.575$.
  - p-value $= 0.005012$.
  - $\overline{x} = 12.528$.
  - $s = 4.740$ ($\sigma = 4.8$).

Example

- Work Example 10.1 on page 616, assuming that σ is unknown.
- Step 1: State the hypotheses.

$$H_0 : \mu = 15 \text{ mg}$$

$$H_1 : \mu < 15 \text{ mg}$$

- Step 2: State the significance level. $\alpha = 0.05$.
- Step 3: What is the test statistic?
- We must digress.

What if σ is Unknown?

- Let us assume that the population is normal or nearly normal.
- Then the distribution of $\overline{x}$ is normal for all sample sizes n,

$$z = \frac{\overline{x} - \mu}{\sigma/\sqrt{n}}$$

What if σ is Unknown?

- Furthermore, for large n ($n \geq 30$),

$$Z \approx \frac{\overline{x} - \mu}{s/\sqrt{n}}$$

- However, for small n, $\frac{\overline{x}-\mu}{s/\sqrt{n}}$ is not approximately standard normal.

What if σ is Unknown?

- Why not?
- And if it is not $N(0, 1)$, then what is it?

Student's t Distribution

Definition (Student's t distribution)

If the population is normal, then the distribution of the statistic

$$t = \frac{\overline{x} - \mu}{s/\sqrt{n}}$$

is Student's t distribution.

Student's t Distribution

- The t distribution was discovered by W. S. Gosset in 1908.

Wolfram MathWorld Article

Student's t-Distribution

The *t* Distribution

- The shape of the *t* distribution is very similar to the shape of the standard normal distribution.
- It is
  - Symmetric
  - Unimodal
  - Centered at 0.
- But it is wider than the standard normal.
- That is because of the additional variability introduced by using *s* instead of σ.

The t Distribution

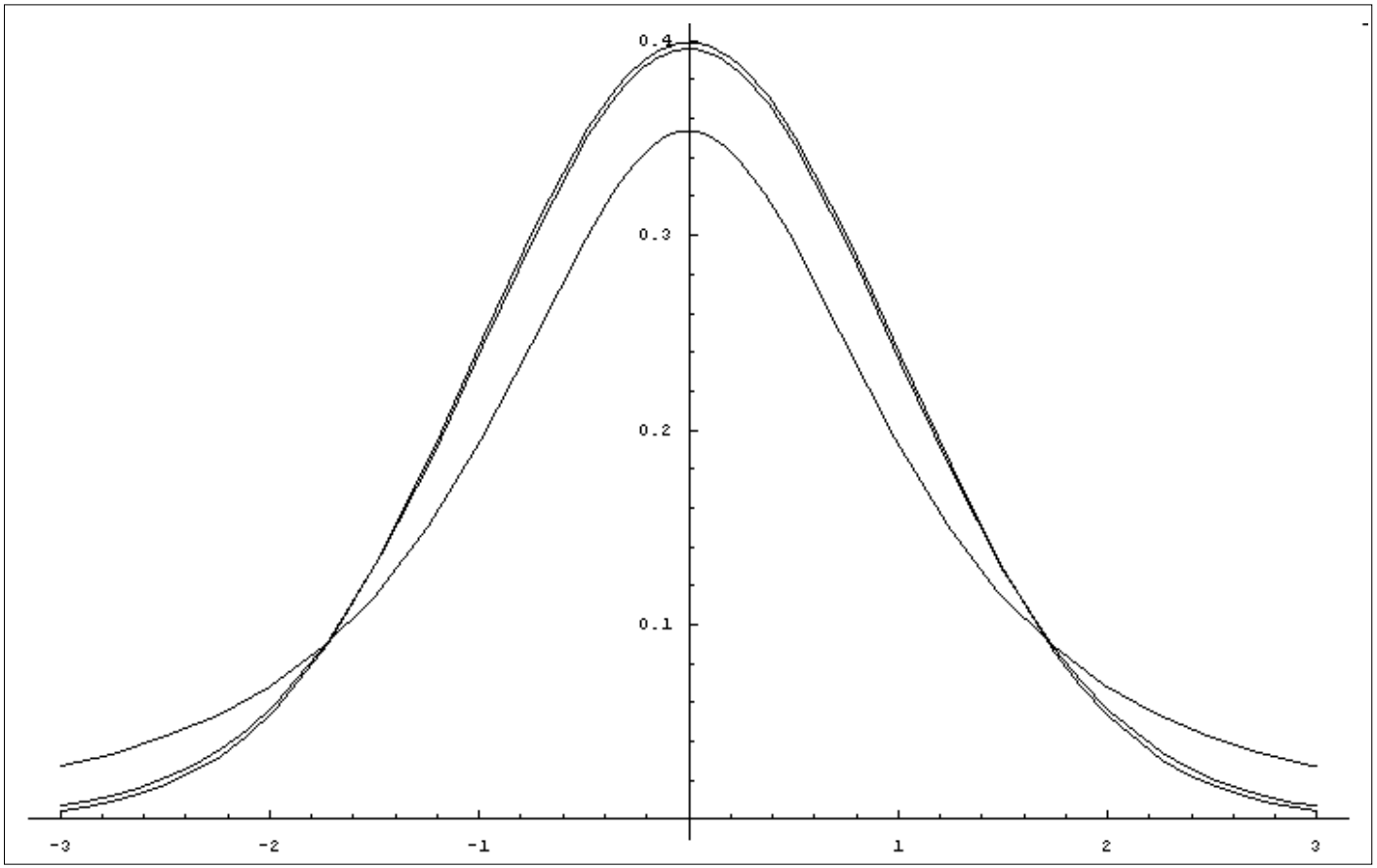
- Furthermore, the t distribution
  - has a slightly different shape for each possible sample size.
  - approaches the standard normal as n gets larger and larger.
- In fact, if $n \geq 30$, then the t distribution is approximately standard normal.

Degrees of Freedom

- If the sample size is n, then t is said to have $n - 1$ degrees of freedom.
- We use df to denote "degrees of freedom."
- We will use the notation t_5 to denote the t distribution with 5 degrees of freedom (i.e., sample size 6).

Standard Normal vs. t Distribution

The distributions t_2, t_{30}, and Z.



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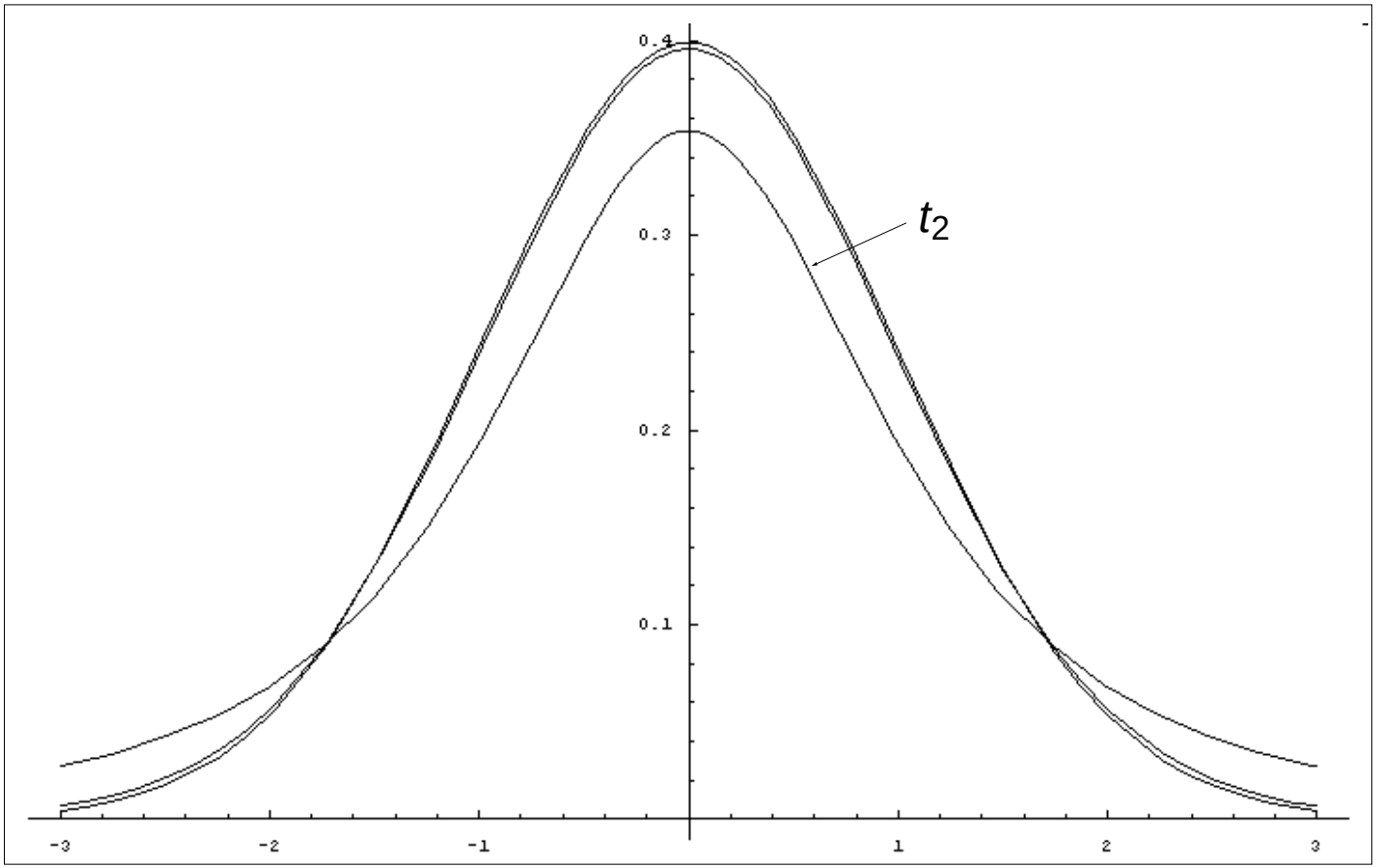

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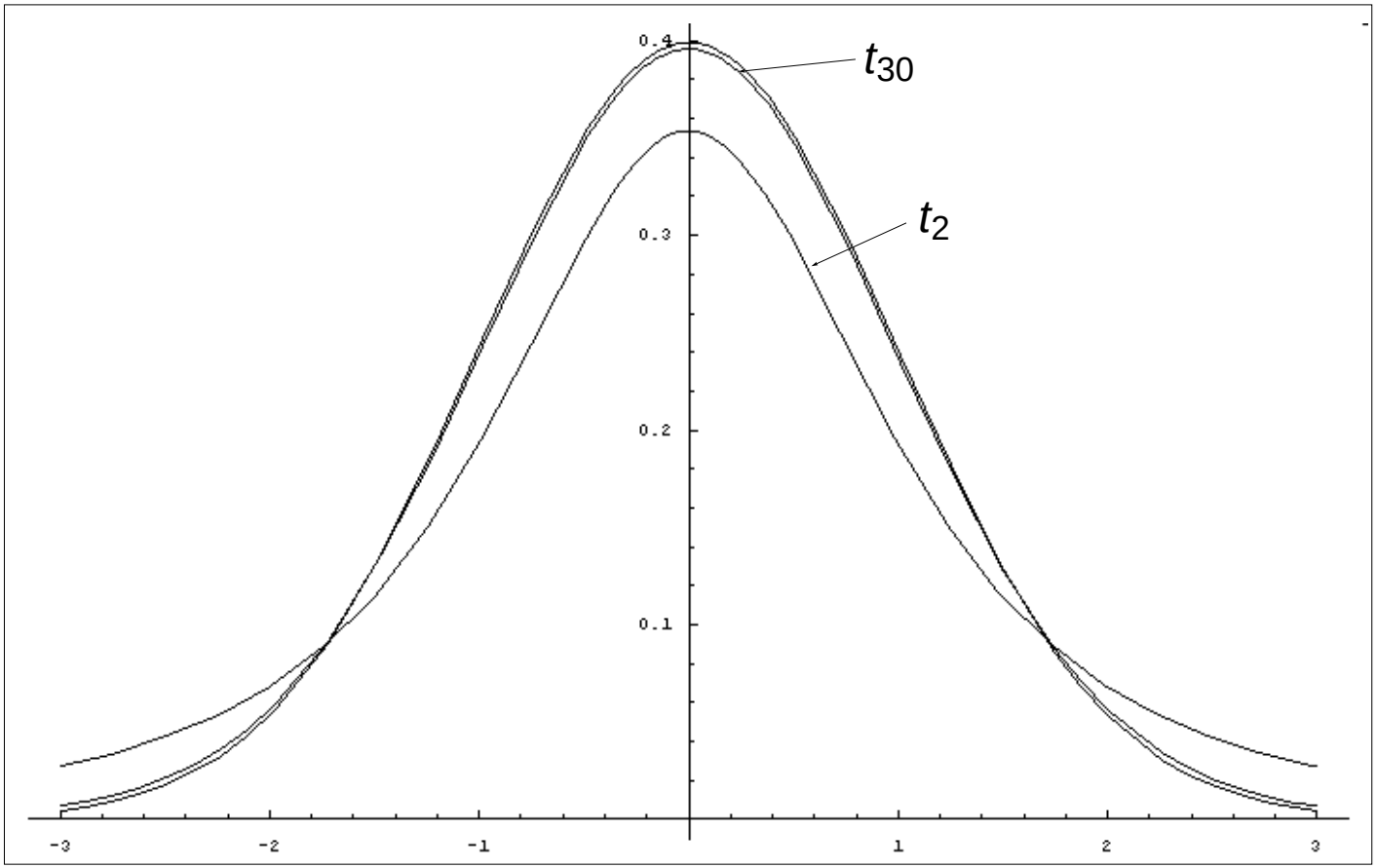

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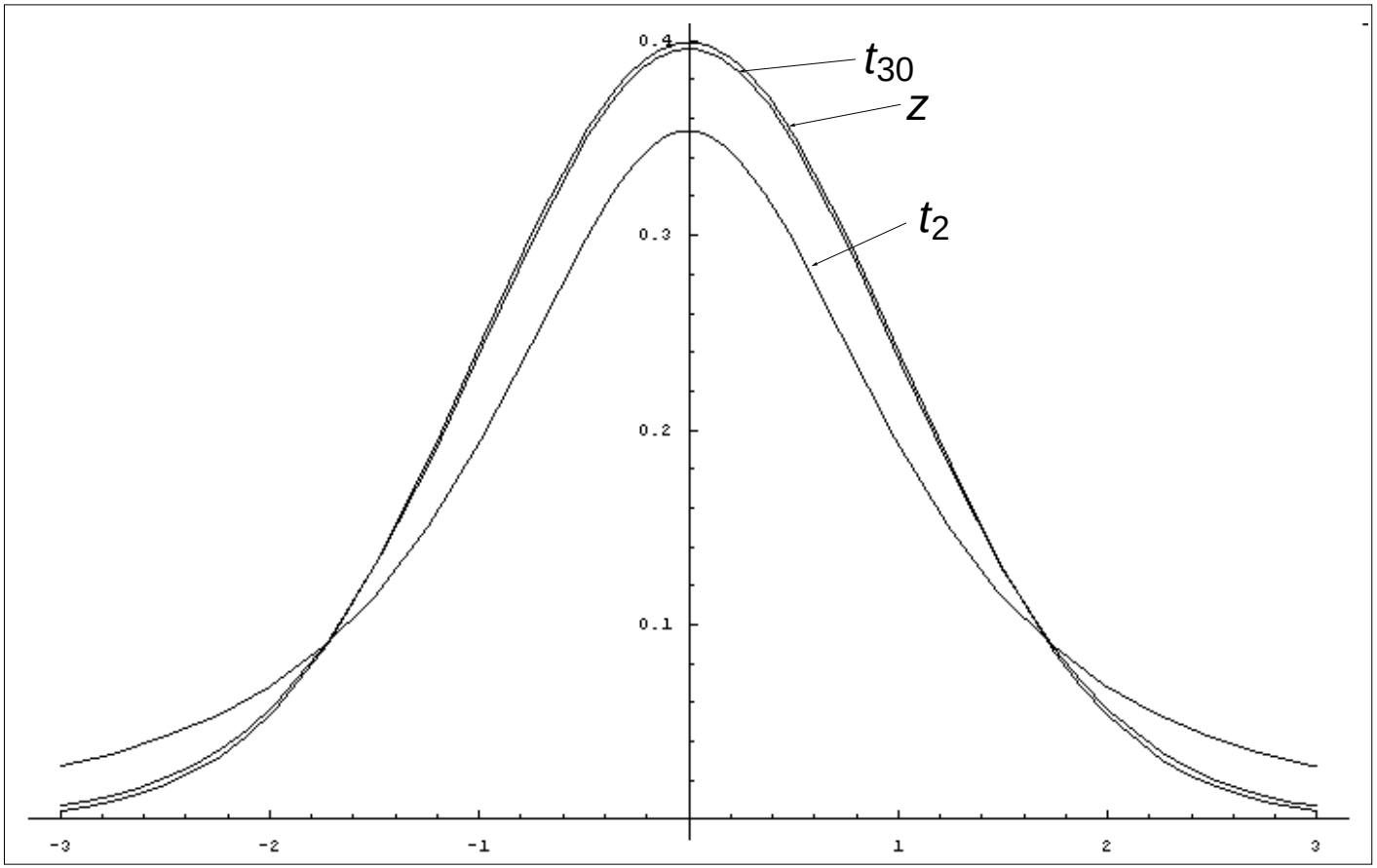
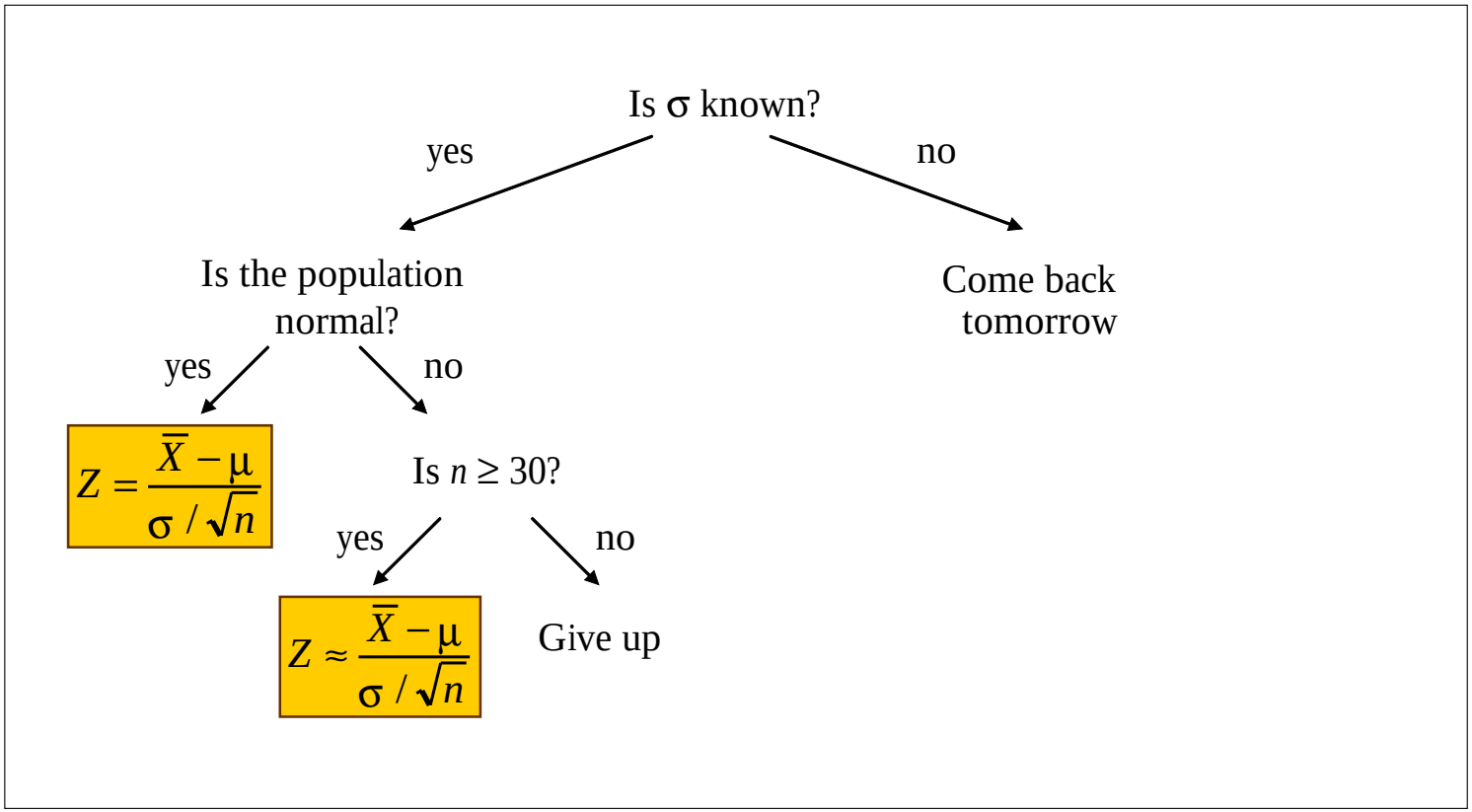


The Decision Tree



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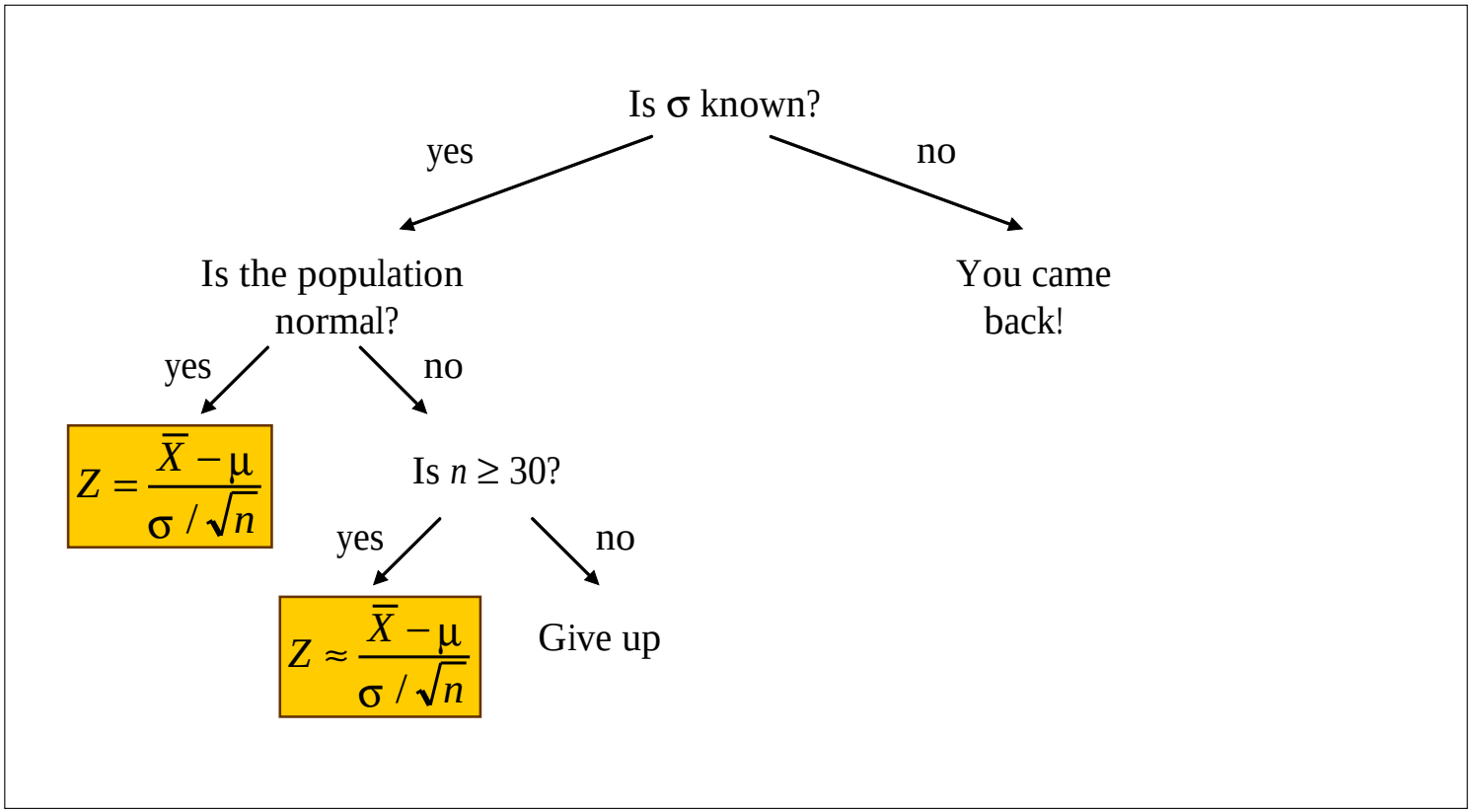

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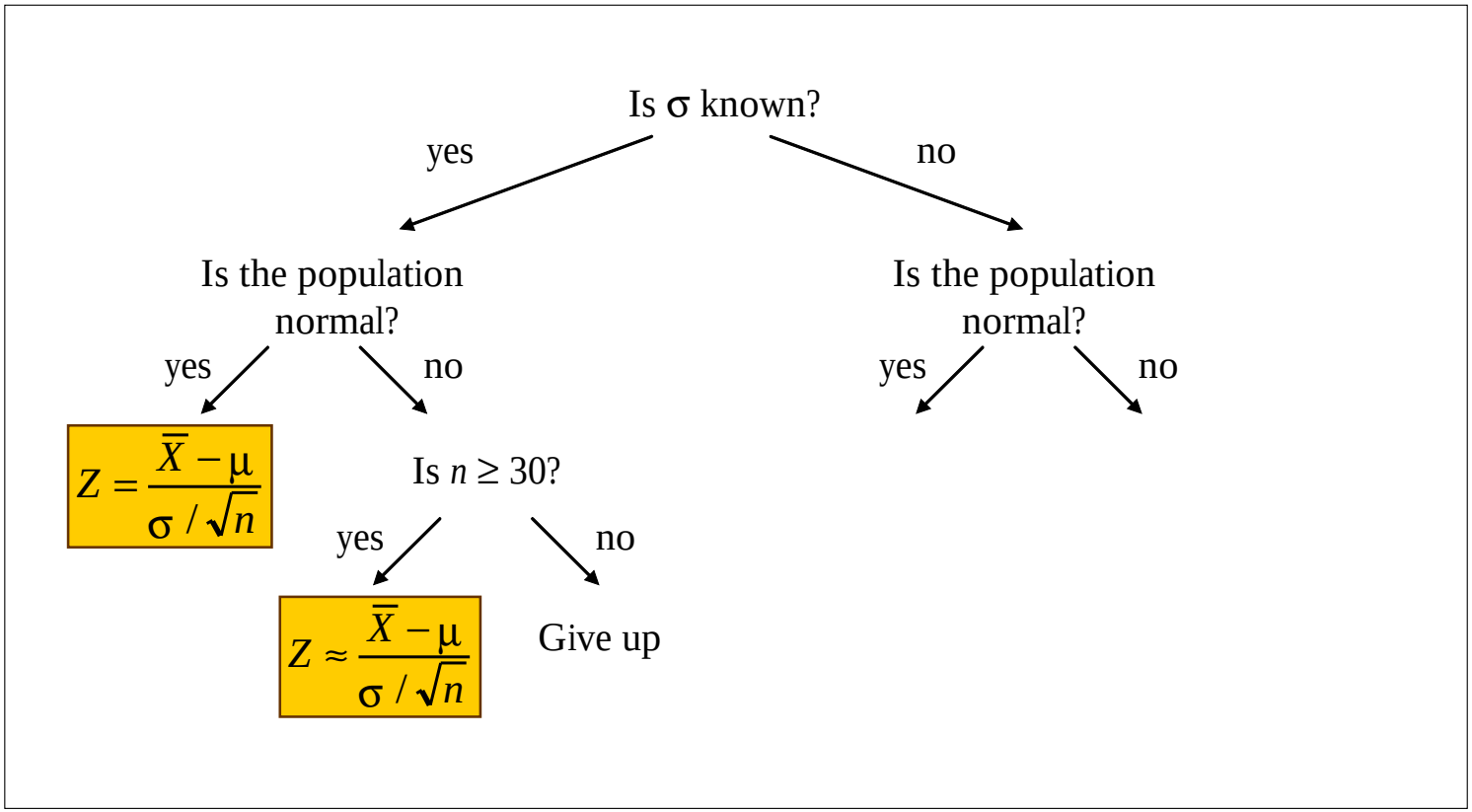

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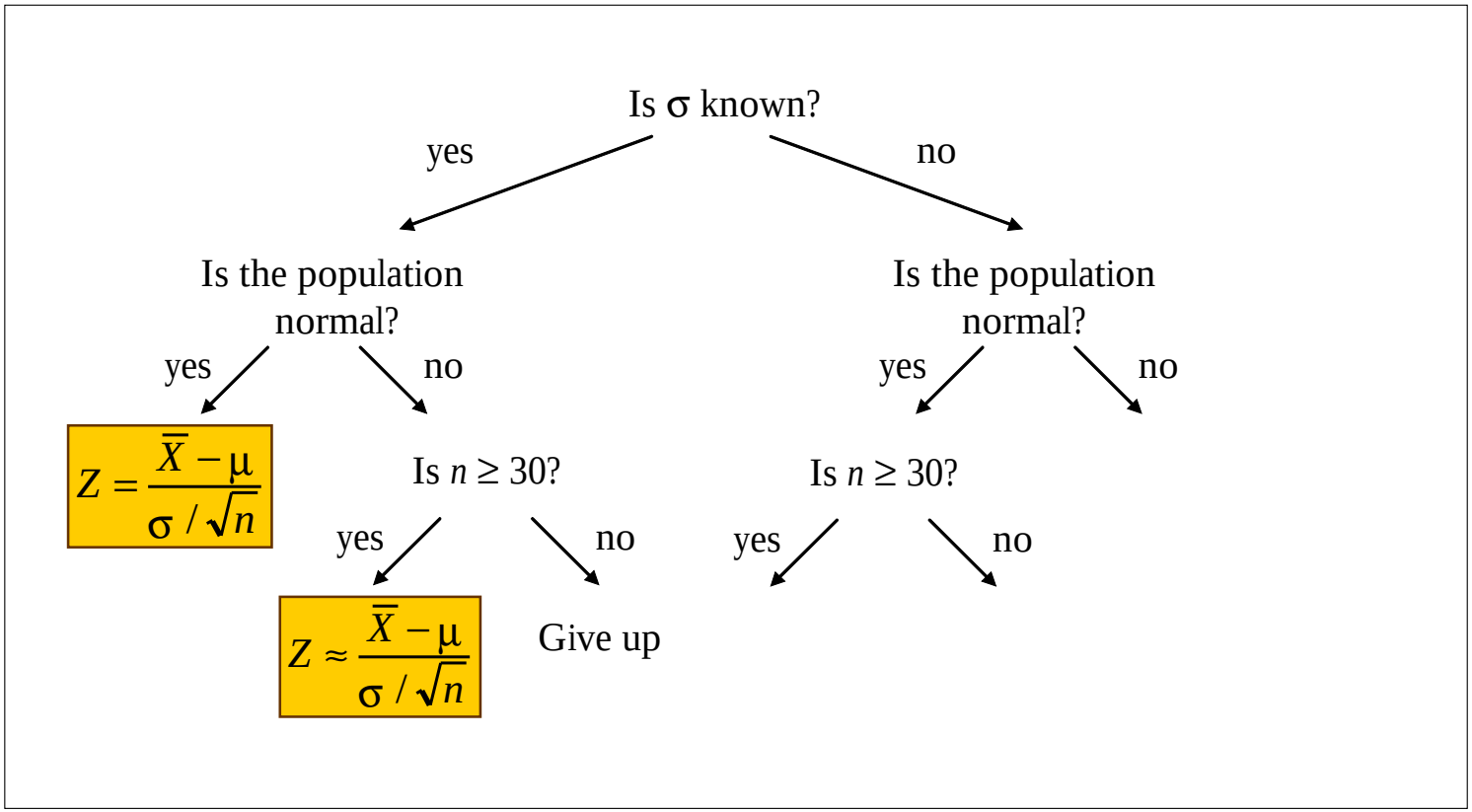

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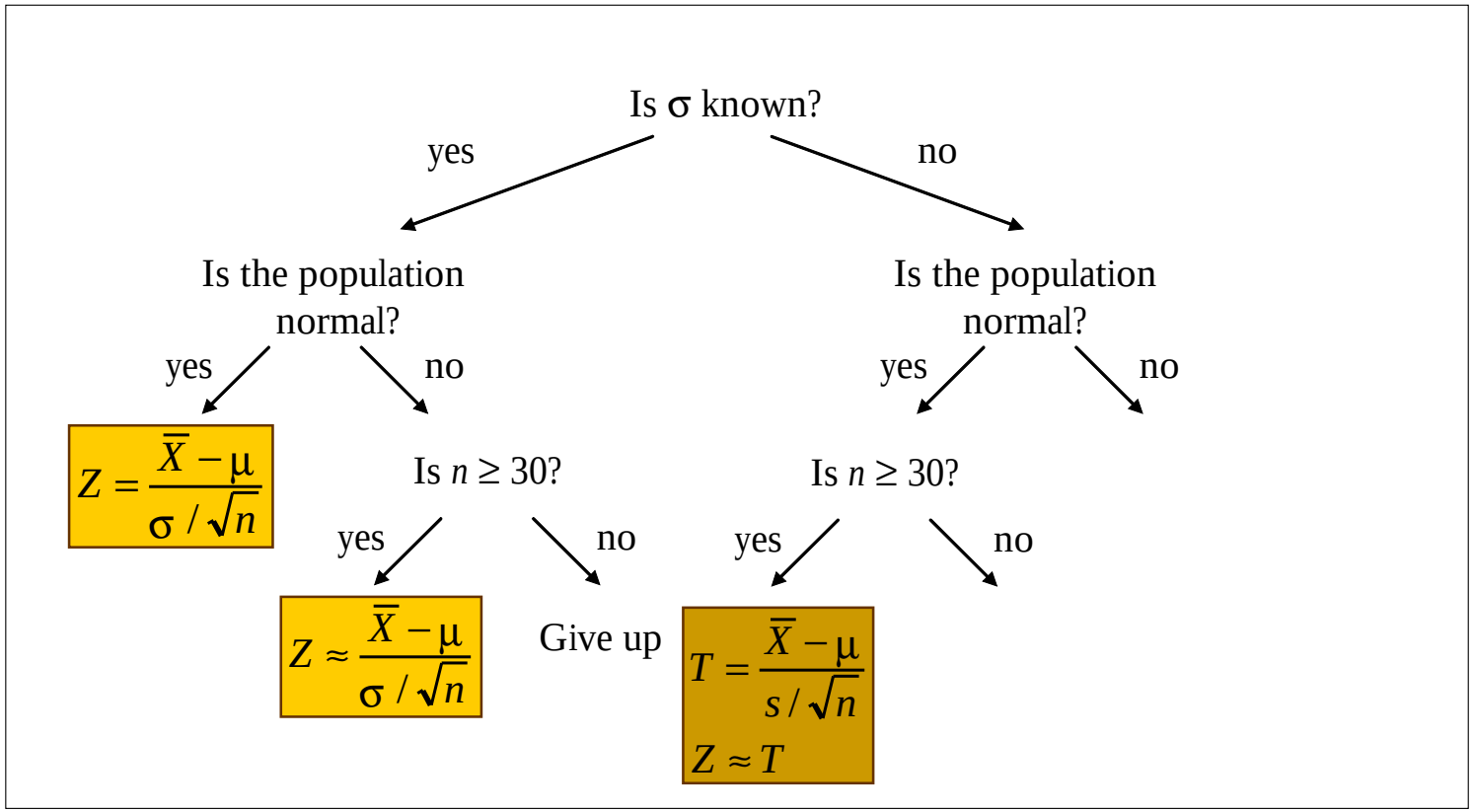

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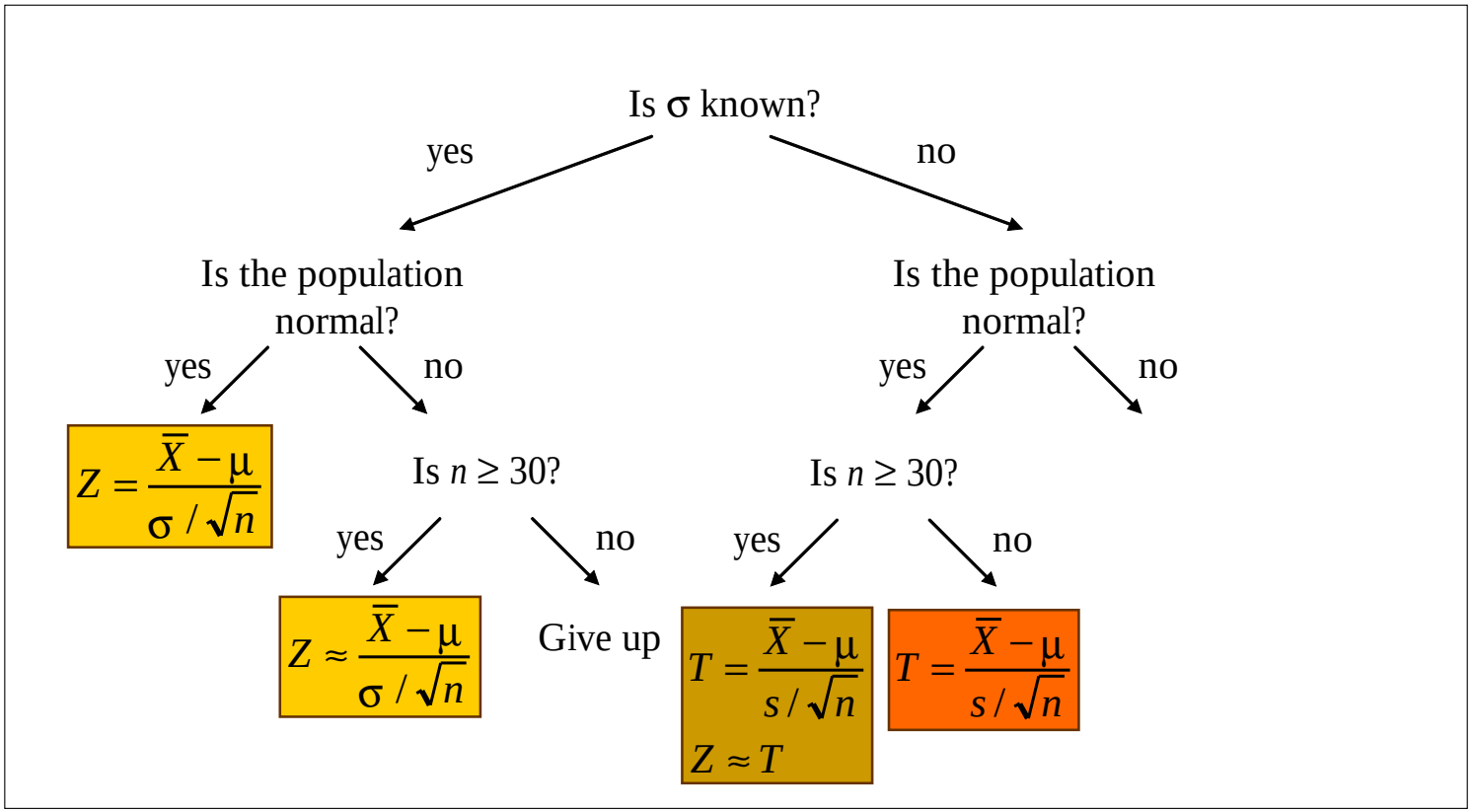

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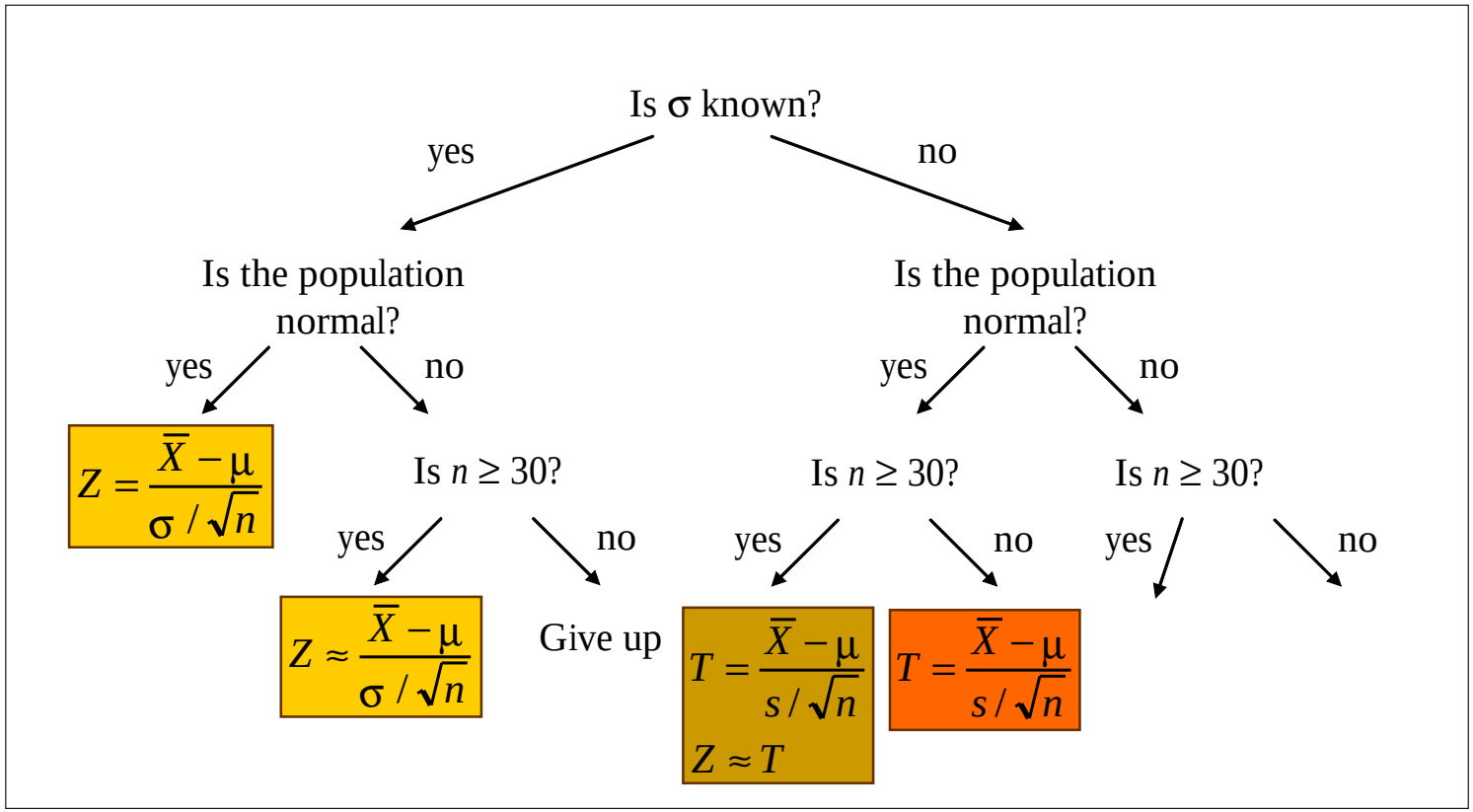

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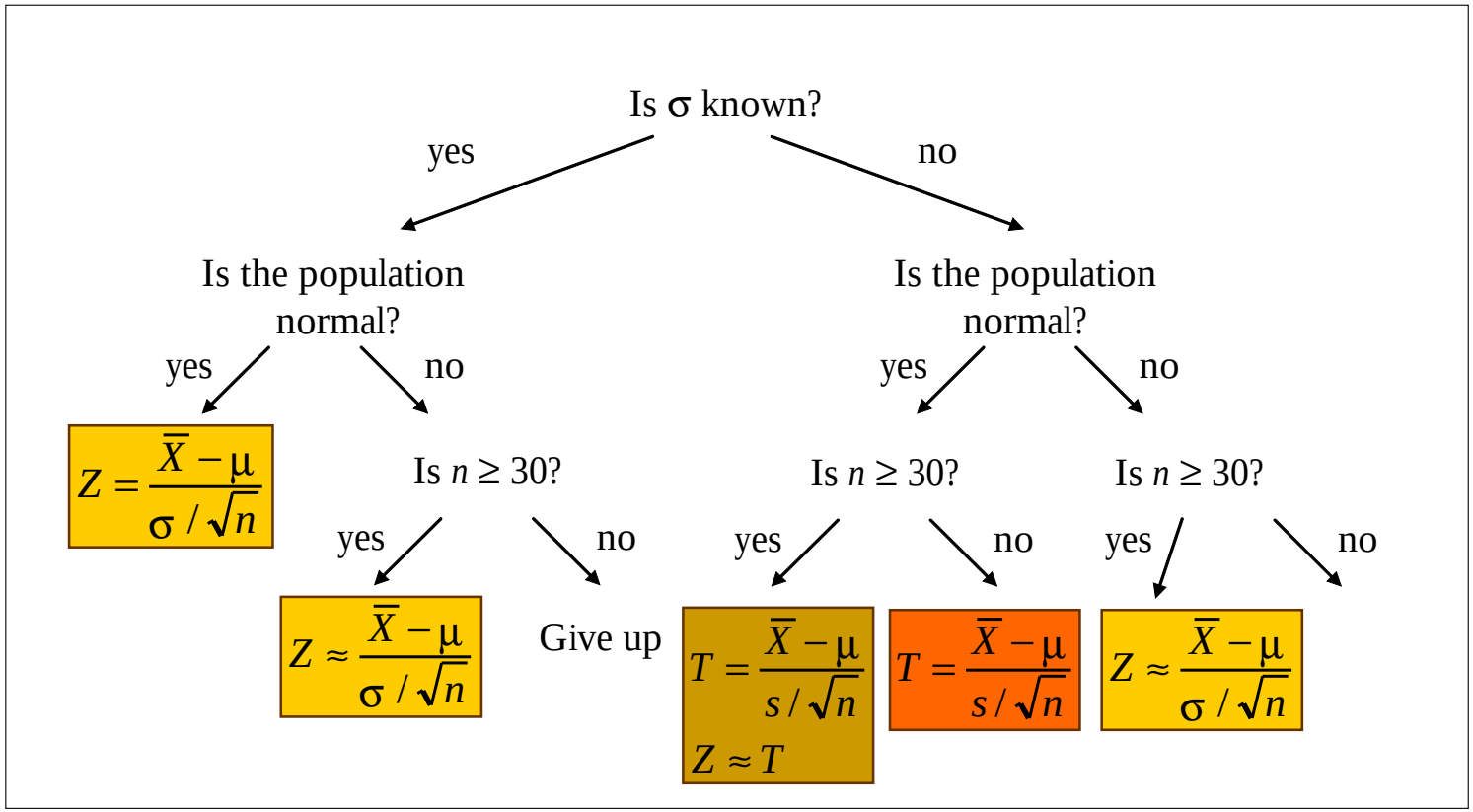

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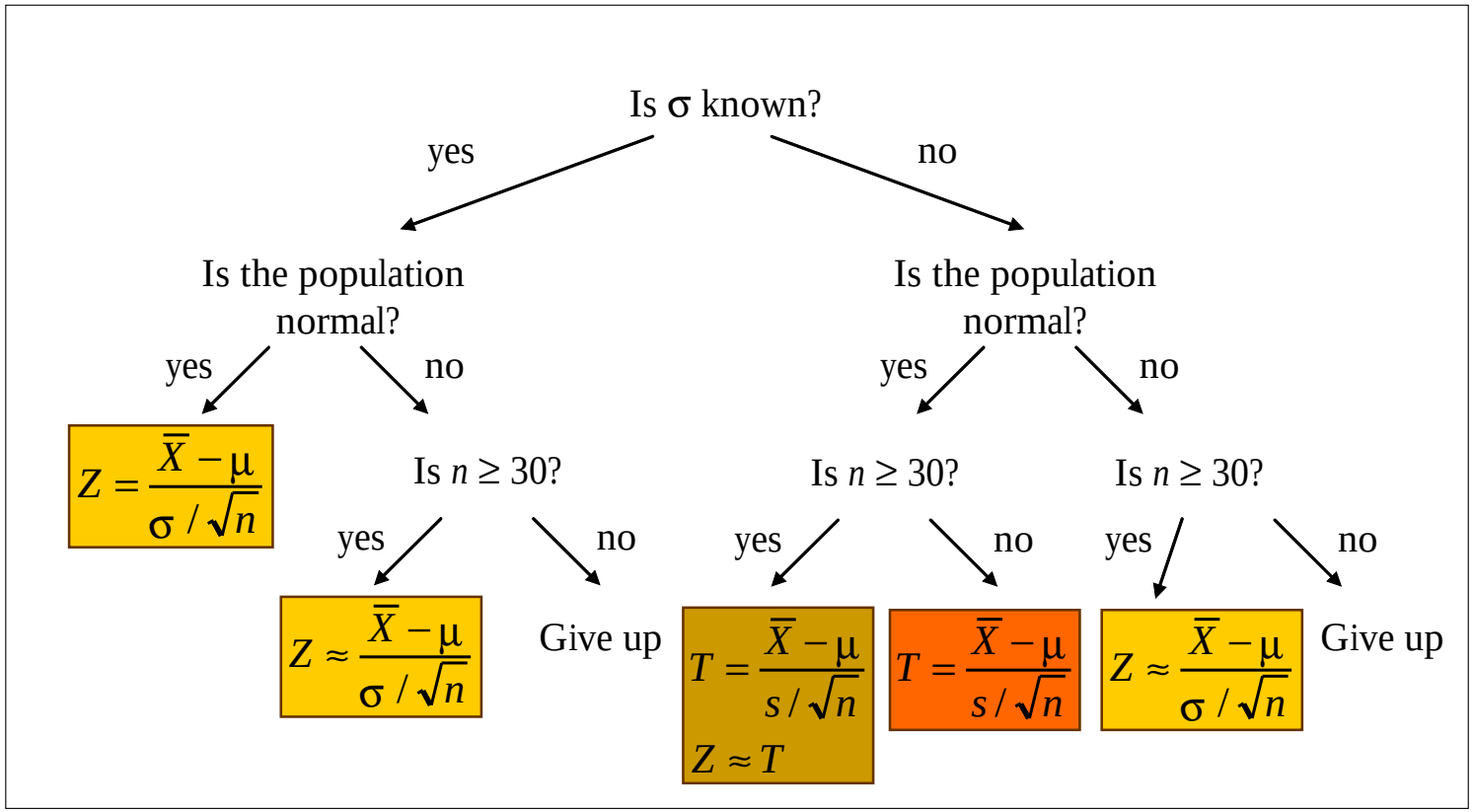

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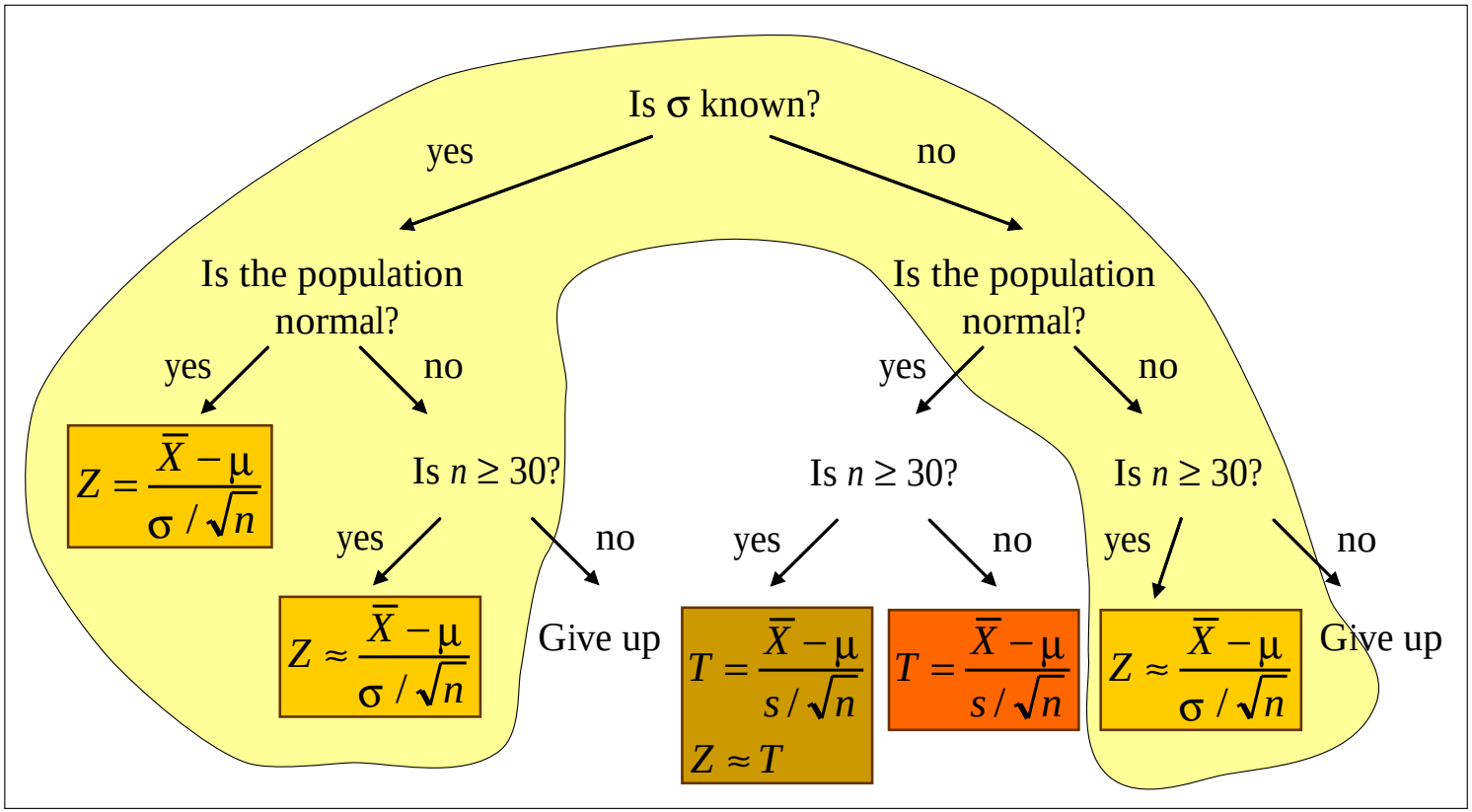

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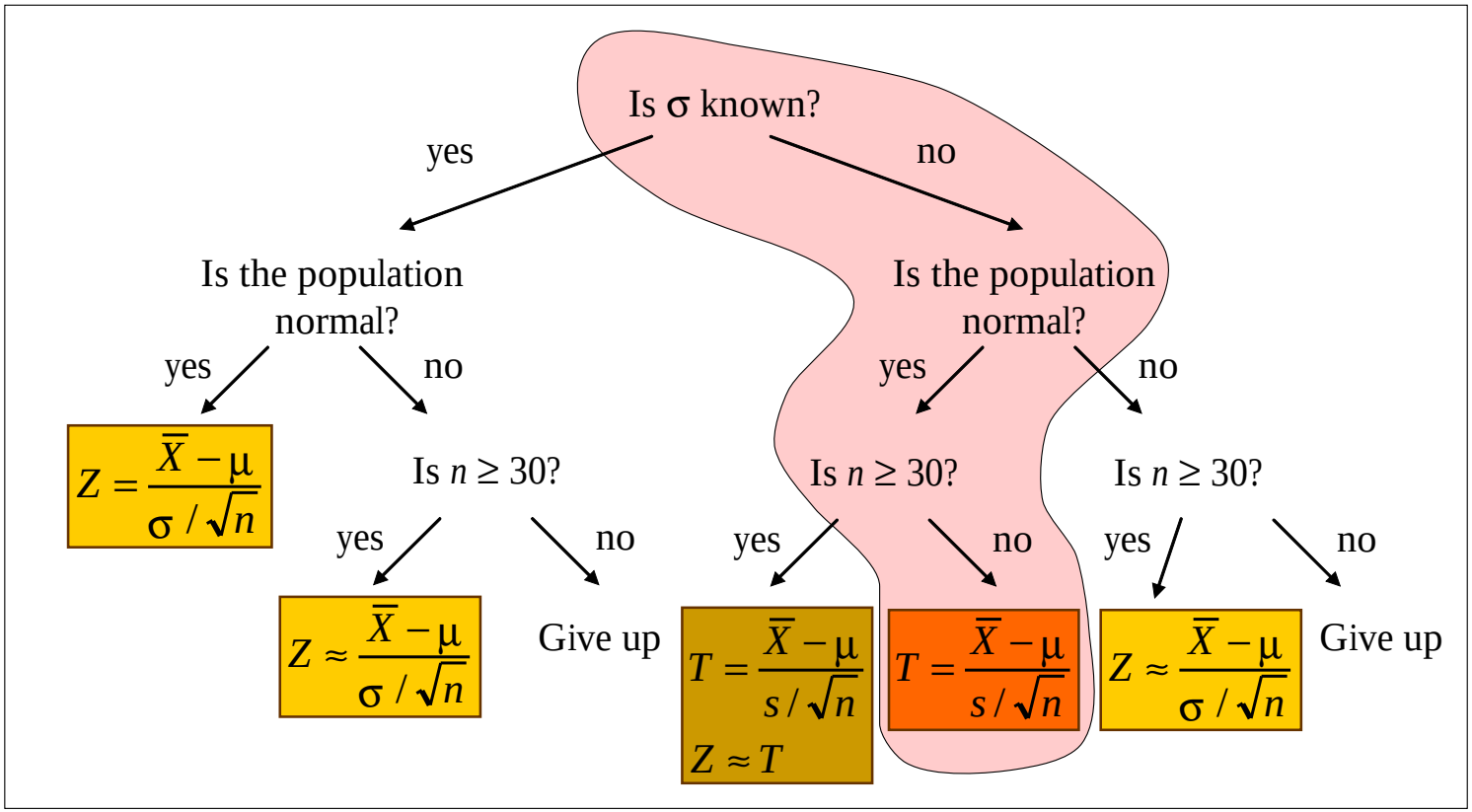

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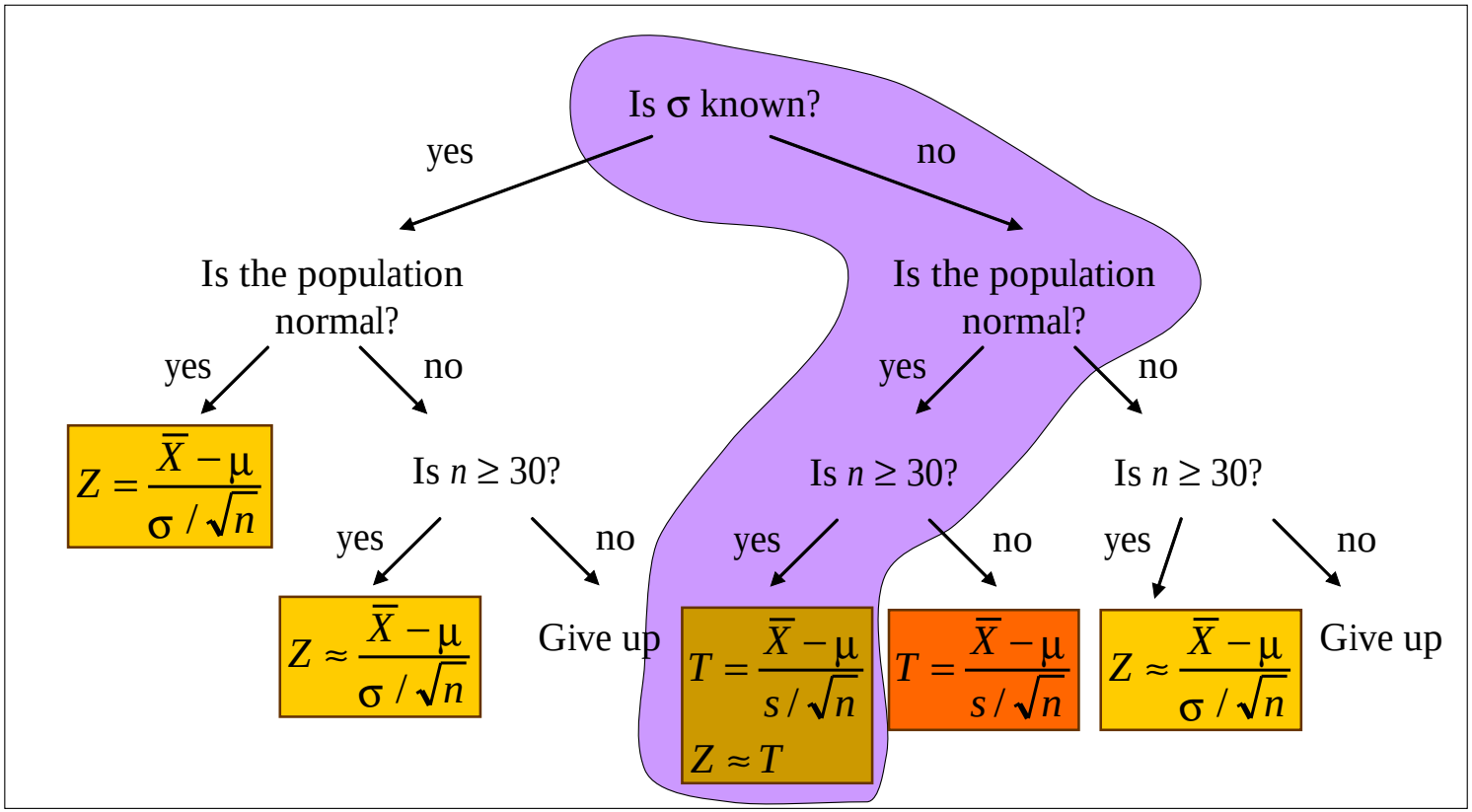

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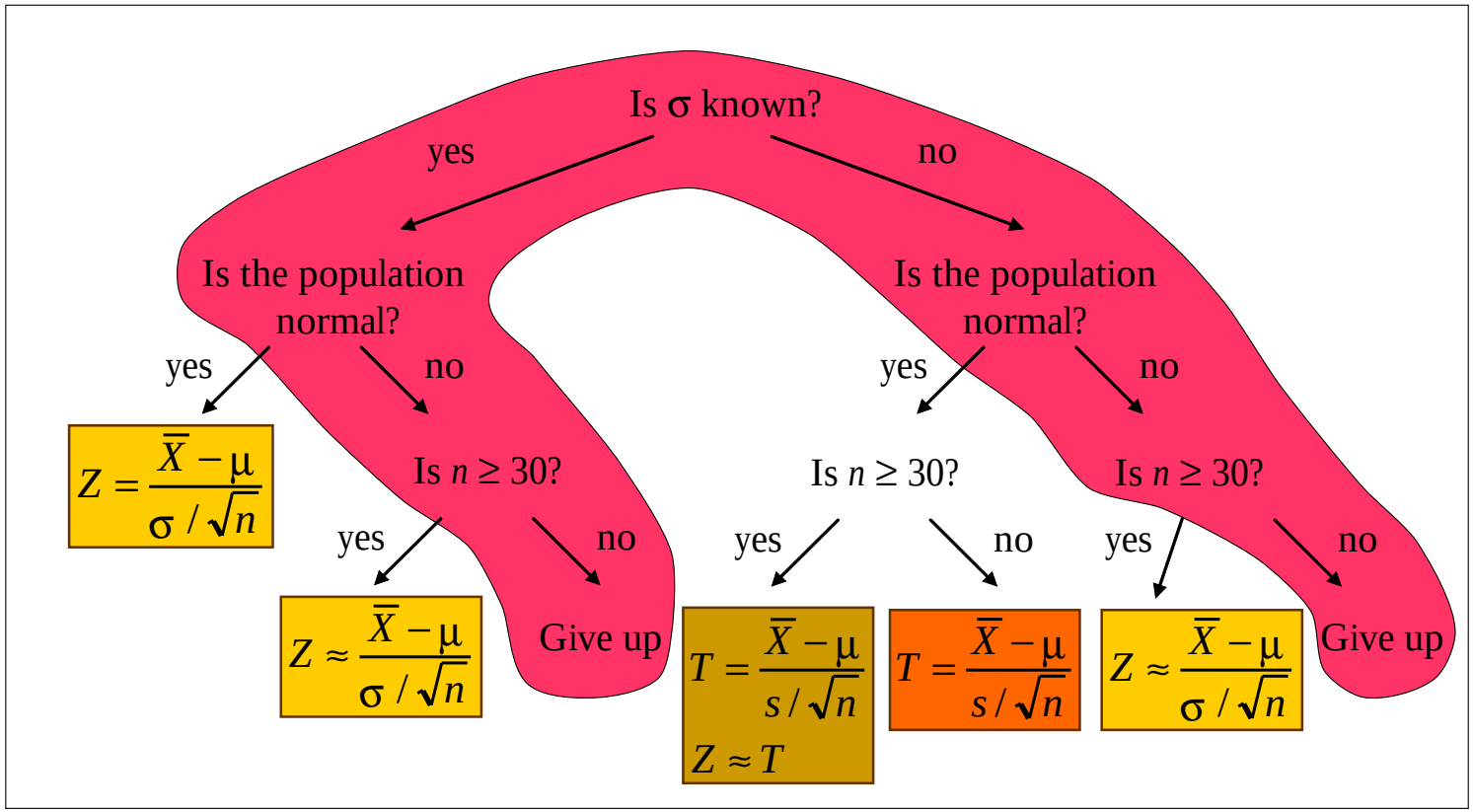

When to Use Z

- Use Z whenever
  - The sample size is large ($n \geq 30$), or
  - The population is normal and σ is known.

When to use t

- Use t when
  - The population is normal, and
  - σ is not known, and (optionally)
  - The sample size is small.

When to Give Up

- Give up when
  - The population is not normal, and
  - The sample size is small ($n < 30$).

Summary

- The TI-83 function ZTest will perform a hypothesis test for the mean when σ is known.
- If σ is not known, then we use s in its place.
- In this case, we may have to use the t test instead of the Z test.
- Student's t distribution is similar to the standard normal distribution, except that it is wider.
- As the number of degrees of freedom increases, the t distribution approaches the standard normal distribution.