Central Limit Theorem Examples

Lecture 28
Sections 8.2, 8.4

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Outline

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The Central Limit Theorem for Means

- The Central Limit Theorem for Means describes the distribution of $\overline{x}$ in terms of μ, σ, and n.
- A problem may ask about a single observation, or it may ask about the sample mean in a sample of observations.
- If it asks about a single observation, then do *not* try to use the Central Limit Theorem.
- However, if it asks about a sample mean, then you *must* use the Central Limit Theorem.

Outline

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Example

Exercise 8.37, page 558.

- A population consists of the three numbers: 1, 5, 9.
- A random sample of two is taken *with* replacement, the two observations being X_1 and X_2.
- The accompanying table (next slide) provides a partial listing of the possible random samples of size 2.
- Let $\overline{X}$ be the sample mean of the two selected observations,

$$\overline{X} = \frac{X_1 + X_2}{2}.$$

Example

Exercise 8.37, page 558.

X_1	X_2	$\overline{X}$
1	1	
1	5	
1	9	
5	1	
5	5	
5	9	

Example

Exercise 8.37, page 558.

(a) Complete the table by listing the remaining possible random samles of size 2 and filling the $\overline{X}$ column for all possible random samples.
(b) Find the exact probability $P(\overline{X} = 5)$.
(c) What is the mean or expected value of $\overline{X}$ (Find the mean of all the sample means values.)

Outline

Applications

Example 8.10, page 540.

Let X be the length of a pregnancy in days. Thus, X is a continuous random variable. Suppose that it has approximately a normal distribution with a mean of 266 days and a standard deviation of 16 days.

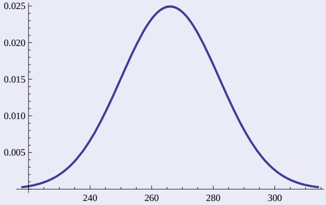
(a) Sketch the distribution for X.

Applications

Example 8.10, page 540.

Let X be the length of a pregnancy in days. Thus, X is a continuous random variable. Suppose that it has approximately a normal distribution with a mean of 266 days and a standard deviation of 16 days.

(a) Sketch the distribution for X.



Applications

Example 8.10, page 540.

(b) Suppose that we have a random sample of $n = 25$ pregnant women. Describe the sampling distribution of $\overline{x}$.

Applications

Example 8.10, page 540.

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It is normal with mean 266 and standard deviation

$$\frac{16}{\sqrt{25}} = 3.2.$$

Applications

Example 8.10, page 540.

(c) What is the probability that a randomly selected pregnancy lasts more than 274 days?

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`normalcdf(274,E99,266,16)` $= 0.3086$.

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(d) Is it more likely or less likely, as compared to part (c), that we might observe a sample mean pregnancy length of more than 274 days? Explain.

Applications

Example 8.10, page 540.

(c) What is the probability that a randomly selected pregnancy lasts more than 274 days?

`normalcdf(274,E99,266,16)` $= 0.3086$.

(d) Is it more likely or less likely, as compared to part (c), that we might observe a sample mean pregnancy length of more than 274 days? Explain.

The probability is `normalcdf(274,E99,266,3.2)` $= 0.0062$, which is much less. That is because the standard deviation is smaller, pulling the values of $\overline{x}$ closer to the mean.

Example

Exercise 8.43, page 560.

In human engineering and product design, it is important to consider the weights of people so that airplanes or elevators aren't overloaded. Based on data from the National health Survey, the weight for adult males in the United States follows a normal distribution with a mean weight of 173 pounds and a standard deviation of 30 pounds.

(a) If one U.S. adult male is randomly selected, what is the probability that his weight will be greater than 180 pounds?

Example

Exercise 8.43, page 560.

In human engineering and product design, it is important to consider the weights of people so that airplanes or elevators aren't overloaded. Based on data from the National health Survey, the weight for adult males in the United States follows a normal distribution with a mean weight of 173 pounds and a standard deviation of 30 pounds.

(a) If one U.S. adult male is randomly selected, what is the probability that his weight will be greater than 180 pounds?

`normalcdf(180,E99,173,30)` $= 0.4077$.

Example

Exercise 8.43, page 560.

(a) If 36 different U.S. adult males are randomly selected, what is the probability that their sample mean weight will be greater than 180 pounds?

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The distribution of $\overline{x}$ is normal with mean 173 and standard deviation $\sigma_{\overline{x}} = \frac{30}{\sqrt{36}} = 5$.

Example

Exercise 8.43, page 560.

(a) If 36 different U.S. adult males are randomly selected, what is the probability that their sample mean weight will be greater than 180 pounds?

The distribution of $\overline{x}$ is normal with mean 173 and standard deviation $\sigma_{\overline{x}} = \frac{30}{\sqrt{36}} = 5$.

Therefore, the probability that $\overline{x}$ is greater than 180 is `normalcdf(180,E99,173,5)` $= 0.0808$.

Outline

Example

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A manufacturer of a particular model of car claims that the average gas mileage for that model is 34 mpg. A consumer group suspects that the gas mileage is not that high. They believe that it is only 32 mpg. The standard deviation is assumed to be 3 mpg. An independent researcher hopes to settle the question by testing a sample of 64 vehicles of that model. He will test the hypotheses

$$H_0 : \mu = 34$$
$$H_1 : \mu = 32$$

He will reject H_0 if $\overline{x} < 33$.

Example

Example

(a) Find α and β for this test.

Example

Example

(a) Find α and β for this test.

(b) What is the *p*-value of 33.5?

Outline

Assignment

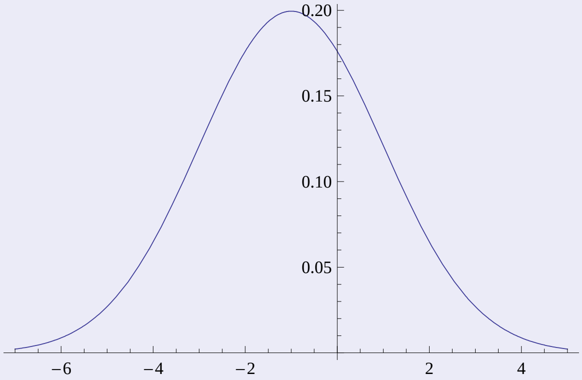
Homework

- Read Section 8.4, pages 531 - 545.
- Let's Do It! 8.10.
- Exercises 24 - 26, 28, page 551.
- Review exercises 33 - 38, 40, 41, 43 - 46, page 557.

Assignment

Answers

24. (a)

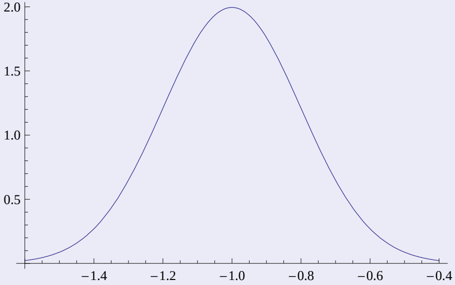


Assignment

Answers

24. (b) (i) It is $N(-1, 0.2)$.

(ii)



(iii) 0.3085.

Assignment

Answers

26. (a) No. Only $1/4$ of the values are less than 4.
    (b) No. The distribution is skewed, so the mean and median are not equal.
    (c) Histogram C. With a sample size of 500, the CLT says that the shape of the distribution will be normal. The other two histograms are clearly not normal.

Assignment

Answers

34. (a) Histogram C.
    (b) (iii)
36. 0.9352.
38. Histogram A.
40. (a) 100.
    (b) 400.
    (c) 1600.
    (d) As the sample size increases, the accuracy of a sample mean increases.

Assignment

Answers

44. (a) No. The probability of such an observation is 0.1056, which is not very small. Therefore, it is not unlikely that it could happen by chance even if the mean really is 750.
    (b) 2 standard deviations of $\overline{x}$.
46. (a) 0.4772.
    (b) 0.4226.