The Normal Distribution

Lecture 20
Section 6.3.1

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Outline

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The Empirical Rule

The Empirical Rule

For any symmetric unimodal distribution,

- Approximately 68% lie within one standard deviation of the mean.
- Approximately 95% lie within two standard deviations of the mean.
- Nearly all lie within three standard deviations of the mean.

The Empirical Rule

Example (Male Heights)

- Suppose that the heights of U.S. adult males are normally distributed with a mean of 69.5 inches and a standard deviation of 2.9 inches.
- Use the Empirical Rule to describe the male heights in more detail.

The Empirical Rule

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- Approx. 68% of males are between 66.6 and 72.4 inches tall.

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- Approx. 68% of males are between 66.6 and 72.4 inches tall.
- Approx. 95% of males are between 63.7 and 75.3 inches tall.

The Empirical Rule

Example (Male Heights)

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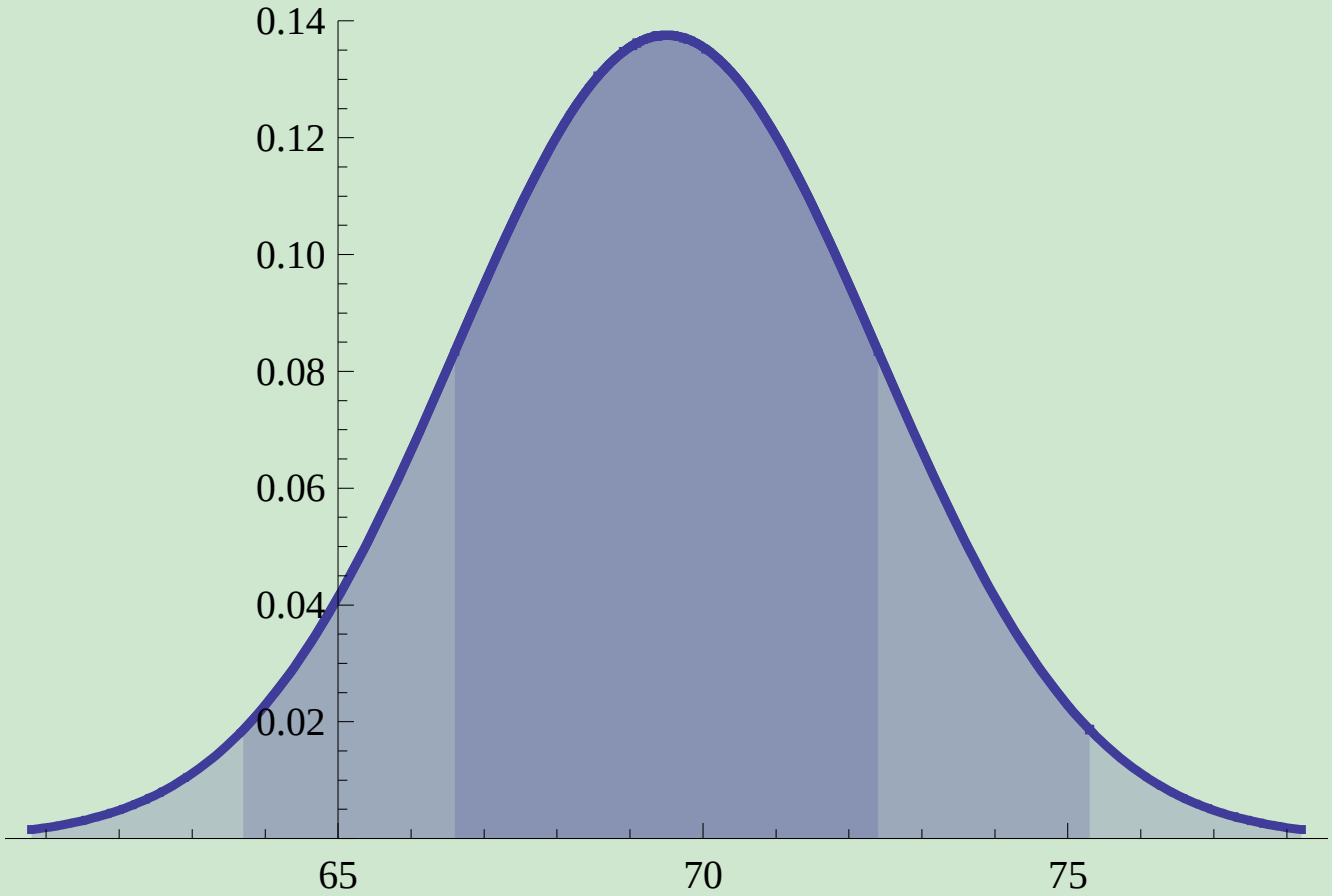
- Use the Empirical Rule to describe the male heights in more detail.
- Approx. 68% of males are between 66.6 and 72.4 inches tall.
- Approx. 95% of males are between 63.7 and 75.3 inches tall.
- Nearly all males are between 60.8 and 78.2 inches tall.

The Empirical Rule

Example (Male Heights)



The normal distribution of male heights

Outline

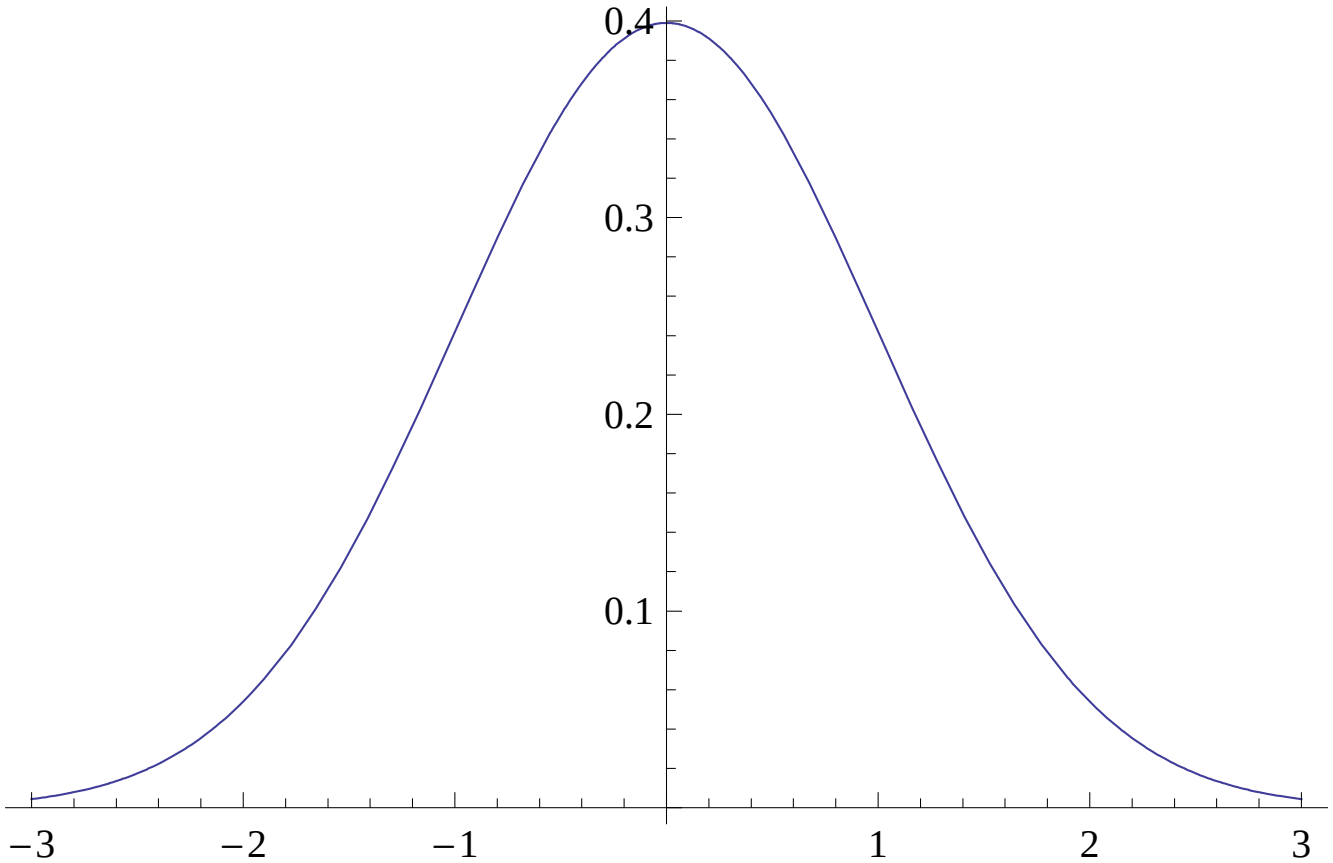
The Standard Normal Distribution

Definition (Standard normal distribution)

The standard normal distribution is the normal distribution with mean 0 and standard deviation 1.

- It is denoted by the letter Z.
- That is, Z is $N(0, 1)$.

The Standard Normal Distribution

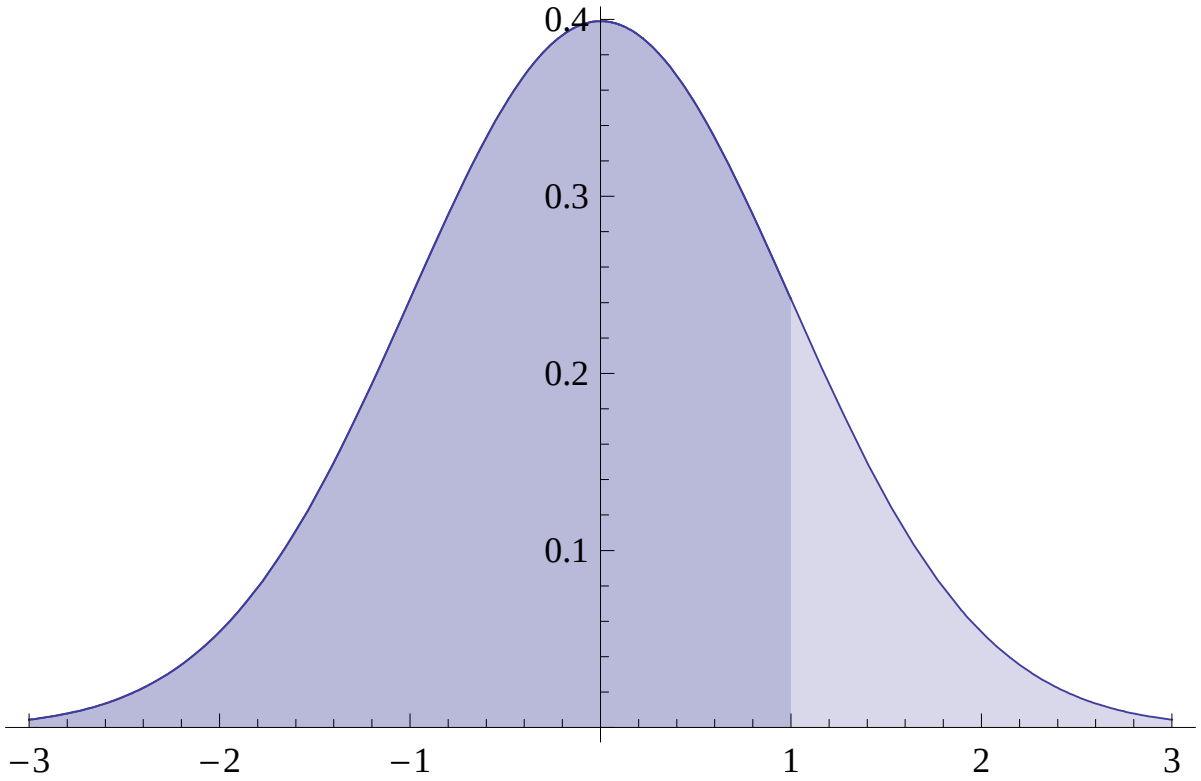


Outline

Areas Under the Standard Normal Curve

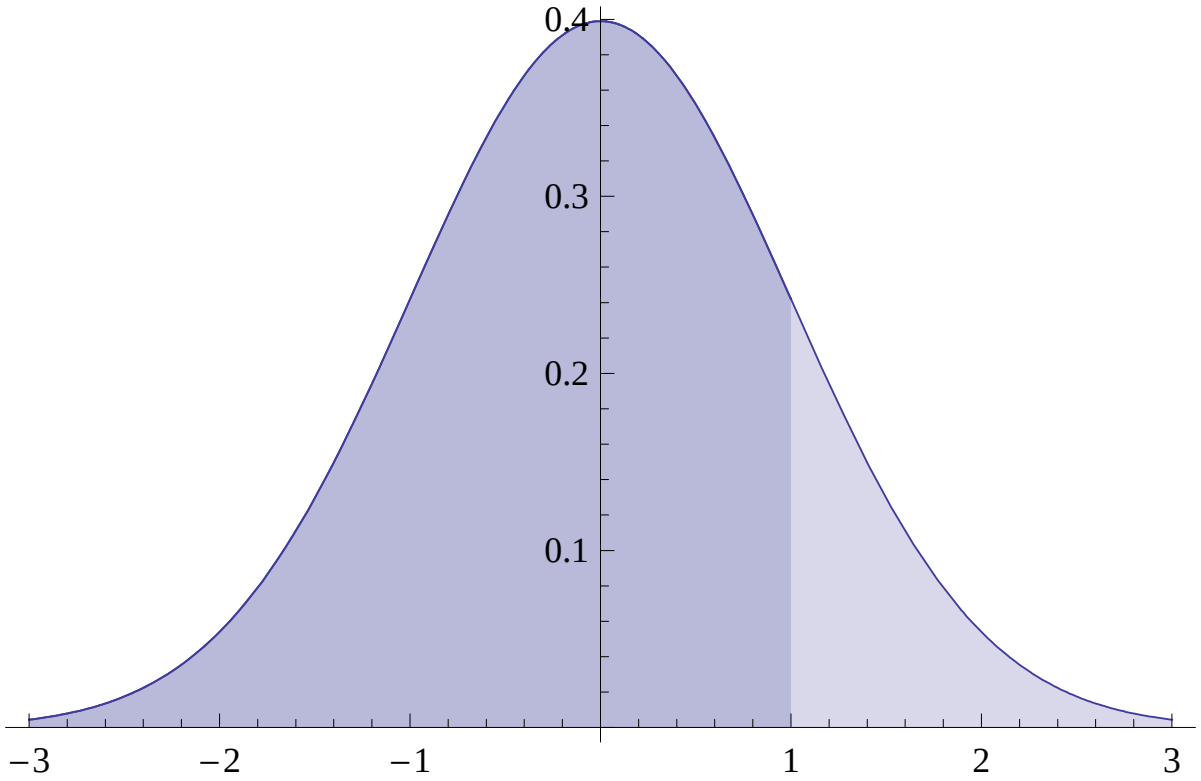
- Easy questions:
  - What is the total area under the curve?
  - What proportion of values of Z will fall below 0?
  - What proportion of values of Z will fall above 0?

Areas Under the Standard Normal Curve



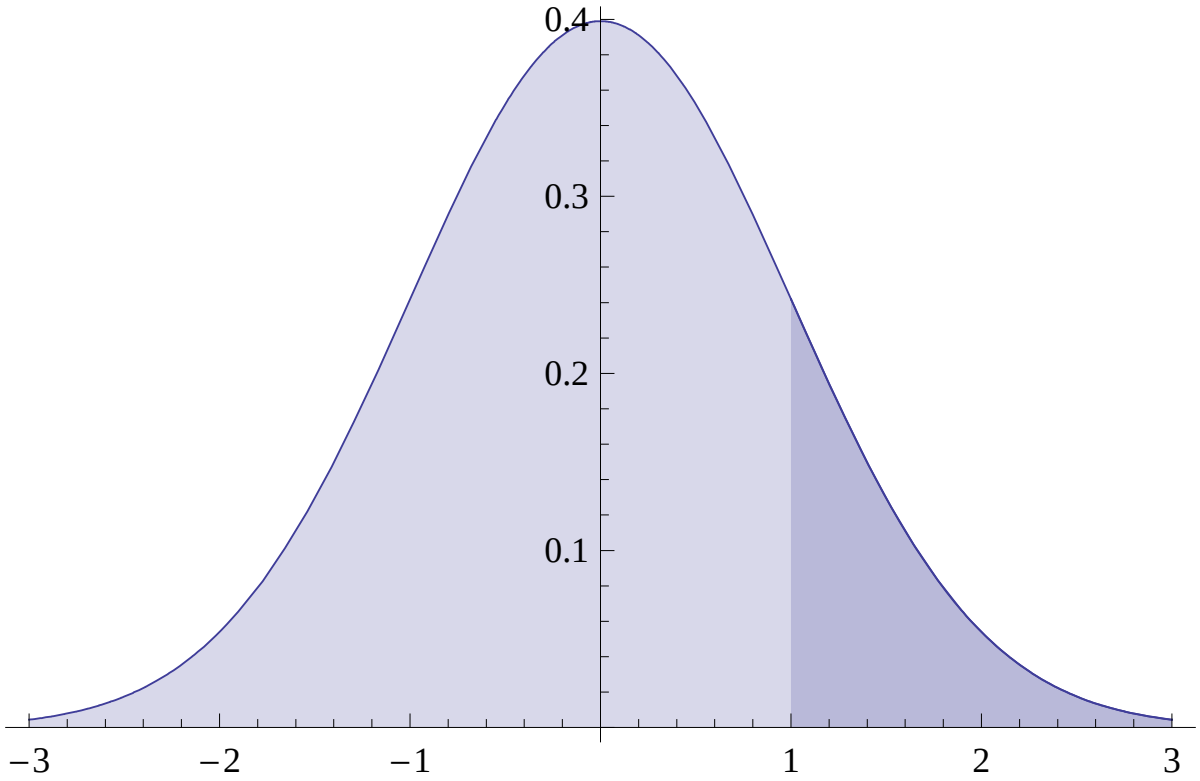
What proportion of values will fall below $+1$?

Areas Under the Standard Normal Curve



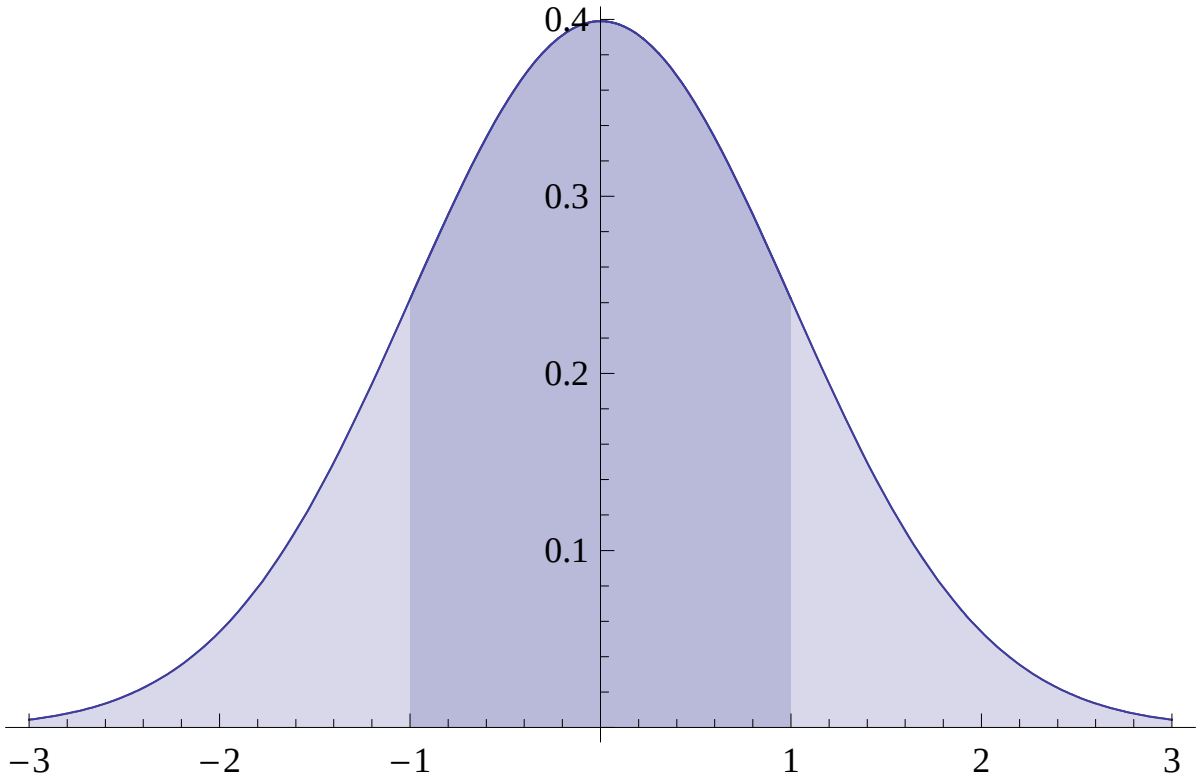
It turns out that the area to the left of $+1$ is 0.8413.

Areas Under the Standard Normal Curve



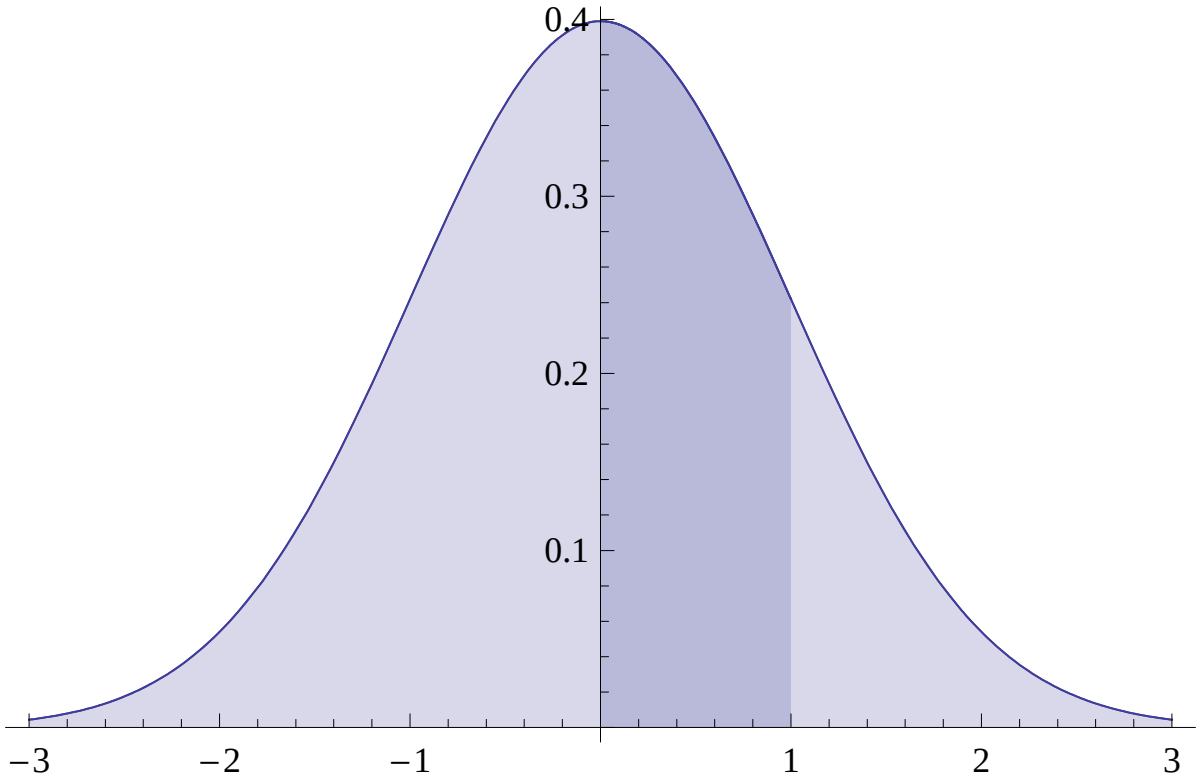
What is the area to the right of $+1$?

Areas Under the Standard Normal Curve



What is the area between -1 and 1?

Areas Under the Standard Normal Curve



What is the area between 0 and 1?

Areas Under the Standard Normal Curve

- There are two methods to finding standard normal areas:
  - The TI-83 function `normalcdf`.
  - Standard normal table.
- We will use the TI-83 (unless you want to use the table).

Outline

TI-83 - Standard Normal Areas

TI-83 Standard Normal Areas

- Press `2nd DISTR`.
- Select `normalcdf` (Item #2).
- Enter the lower and upper bounds of the interval.
  - If the interval is infinite to the left, enter `−E99` as the lower bound.
  - If the interval is infinite to the right, enter `E99` as the upper bound.
- Press `ENTER`. The area appears in the display.

Standard Normal Areas

Practice

- Use the TI-83 to find the following.
  - The area between -1 and 1.
  - The area to the right of 1.
  - The area to the left of 1.645.
- What "standard normal" percentile is 1.645?

Outline

Other Normal Curves

- If we are working with a different normal distribution, say $N(30, 5)$, then how can we find areas under the curve?
- Use the same procedure as before, except enter the mean and standard deviation as the 3rd and 4th parameters of the `normalcdf` function.
- For example, the area between 24 and 36 is given by `normalcdf(24,36,30,5)`.

TI-83 - Area Under Normal Curves

Example (TI-83 Normal Areas)

- Find area between 25 and 38 in the distribution $N(30, 5)$.
- In the TI-83, enter `normalcdf(25,38,30,5)`.
- Press `ENTER`. The answer 0.7865 appears.

Male Heights

- Recall that male heights are $N(69.5, 2.9)$.

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- What proportion of males are less than 66 inches tall?

Male Heights

- Recall that male heights are $N(69.5, 2.9)$.
- What proportion of males are between 65 and 72 inches tall?
- What proportion of males are less than 66 inches tall?
- What proportion of males are at least 6 feet tall?

Male Heights

- Recall that male heights are $N(69.5, 2.9)$.
- What proportion of males are between 65 and 72 inches tall?
- What proportion of males are less than 66 inches tall?
- What proportion of males are at least 6 feet tall?
- If we choose one male at random from the population, what is the probability that he is between 66 and 68 inches tall?

Male Heights

- Recall that male heights are $N(69.5, 2.9)$.
- What proportion of males are between 65 and 72 inches tall?
- What proportion of males are less than 66 inches tall?
- What proportion of males are at least 6 feet tall?
- If we choose one male at random from the population, what is the probability that he is between 66 and 68 inches tall?
- Suppose that to be an undercover agent, you have to be in the middle 70% in height. How short can you be and how tall can you be?

Outline

Assignment

Homework

- Read Section 6.3.1, pages 364 - 370.
- Let's Do It! 6.2, 6.3, 6.4, 6.5, 6.6
- Exercises 4 - 9, 11, 12, 15, 16, 18, page 376.