What’s in the Bag?
Lecture 4
Sections 1.4.1-1.4.2

Robb T. Koether
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Tue, Jan 23, 2012
1. Bag A and Bag B

2. Possible Decision Rules
   - Decision Rule #1
   - Decision Rule #2
   - Decision Rule #3

3. $\alpha$ vs. $\beta$

4. Assignment
The results of a study were found not to be statistically significant. We may conclude that
(a) The null hypothesis was accepted.
(b) The null hypothesis was not accepted.
(c) The alternative hypothesis was accepted.
(d) We cannot tell which was accepted.
The results of a study were found not to be statistically significant. We may conclude that

(a) A Type I error could have been made.
(b) A Type II error could have been made.
(c) Either type of error could have been made.
(d) Neither type of error could have been made.
Example (Review Quiz Answers)

1. (a) The null hypothesis was accepted.
2. (b) A Type II error could have been made.
1. Bag A and Bag B

2. Possible Decision Rules
   - Decision Rule #1
   - Decision Rule #2
   - Decision Rule #3

3. $\alpha$ vs. $\beta$

4. Assignment
Two Bags

- Two bags contain vouchers worth various dollar amounts.

Bag A

Bag B

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Two Bags

- The vouchers in Bag A are worth $470.
- The vouchers in Bag B are worth $930.
- You get to cash in the vouchers in whichever bag you want.
- Which bag should you choose?
The vouchers in Bag A are worth $470.
The vouchers in Bag B are worth $930.
You get to cash in the vouchers in whichever bag you want.
Which bag should you choose?
Oh, one more thing...
Two Bags

- The vouchers in Bag A are worth $470.
- The vouchers in Bag B are worth $930.
- You get to cash in the vouchers in whichever bag you want.
- Which bag should you choose?
- Oh, one more thing...
- You are handed one of the bags, but you do not know which bag is it.
Two Bags

- The vouchers in Bag A are worth $470.
- The vouchers in Bag B are worth $930.
- You get to cash in the vouchers in whichever bag you want.
- Which bag should you choose?
- Oh, one more thing...
- You are handed one of the bags, but you do not know which bag is it.
- You may look at one voucher in that bag.
The vouchers in Bag A are worth $470.
The vouchers in Bag B are worth $930.
You get to cash in the vouchers in whichever bag you want.
Which bag should you choose?
Oh, one more thing...
You are handed one of the bags, but you do not know which bag is it.
You may look at one voucher in that bag.
Based on that alone, you must choose which bag to cash in.
The hypotheses are:

- $H_0$: You are holding Bag A.
- $H_1$: You are holding Bag B.

You will choose one voucher at random from the bag and read its value.

From that information, you will decide which bag you believe you are holding.

Then you will choose the one that you believe is Bag B, take the money and run.
### Decisions and Their Consequences

#### What's in the Bag?

<table>
<thead>
<tr>
<th>The State of Nature</th>
<th>Our Decision</th>
</tr>
</thead>
<tbody>
<tr>
<td>It is Bag A</td>
<td>Win $930</td>
</tr>
<tr>
<td>It is Bag B</td>
<td>Win $470</td>
</tr>
</tbody>
</table>

- It is Bag A (switch bags)
- It is Bag B (keep the bag)
1. Bag A and Bag B

2. Possible Decision Rules
   - Decision Rule #1
   - Decision Rule #2
   - Decision Rule #3

3. $\alpha$ vs. $\beta$

4. Assignment
There are several possible decision rules, some better than others.

We will look at three of them.
1 Bag A and Bag B

2 Possible Decision Rules
   • Decision Rule #1
   • Decision Rule #2
   • Decision Rule #3

3 $\alpha$ vs. $\beta$

4 Assignment
A Decision Rule

Decision Rule #1

Reject $H_0$ if the voucher is worth $60$.

- What is $\alpha$?
- What is $\beta$?
Rejection and Acceptance Regions

Definition (Rejection Region)
The rejection region is the set of possible observed values that will lead to rejection of the null hypothesis.

Definition (Acceptance Region)
The acceptance region is the set of possible observed values that will lead to acceptance of the null hypothesis.

Definition (Critical Value(s))
The critical value(s) is the value that separates the the two regions. The critical value itself will be included in the rejection region.
Decision Rule #1

Critical value
Decision Rule #1

![Graph showing the critical value and rejection region. The critical value is indicated at 60, and the rejection region extends to the right.]
Decision Rule #1

- Critical value
- Acceptance region
- Rejection region
Compute $\alpha$
Compute $\alpha$

Bag A

Bag B

What's in the Bag?
Compute $\alpha$

Bag A

$\times$ $\times$ $\times$ $\times$ $\times$ $\times$

$\times$ $\times$ $\times$ $\times$ $\times$ $\times$

$\times$ $\times$ $\times$ $\times$ $\times$ $\times$

(reject $H_0$)

10 20 30 40 50 60

Bag B

$\times$ $\times$ $\times$ $\times$ $\times$ $\times$

$\times$ $\times$ $\times$ $\times$ $\times$ $\times$

$\times$ $\times$ $\times$ $\times$ $\times$ $\times$

10 20 30 40 50 60

(reject $H_0$)
Compute $\alpha$

$\alpha = \frac{1}{20}$

Bag A

Bag B

What's in the Bag?
Compute $\hat{\beta}$

Bag A

Bag B
Compute $\beta$

Bag A

Bag B
Compute $\beta$

Bag A

Bag B

(accept $H_0$)
Compute $\beta$

Bag A

Bag B

$\beta = \frac{13}{20}$
Extreme Values

Definition (Direction of Extreme)

The direction of extreme is the direction, left or right or both, in which we are more likely to reject $H_0$.

- Normally, either
  - The larger the observed value, the more likely we are to reject $H_0$, or
  - The smaller the observed value, the more likely we are to reject $H_0$. 
1 Bag A and Bag B

2 Possible Decision Rules
   • Decision Rule #1
   • Decision Rule #2
   • Decision Rule #3

3 $\alpha$ vs. $\beta$

4 Assignment
Decision Rule #2

Reject $H_0$ if the voucher is worth at least $50$.

- What is $\alpha$?
- What is $\beta$?
Compute $\alpha$ and $\beta$

Bag A

Bag B
1. Bag A and Bag B

2. Possible Decision Rules
   - Decision Rule #1
   - Decision Rule #2
   - Decision Rule #3

3. \( \alpha \) vs. \( \beta \)

4. Assignment
Yet Another Decision Rule

Decision Rule #3
Reject $H_0$ if the voucher is worth at least $40.$

- What is $\alpha$?
- What is $\beta$?
Compute $\alpha$ and $\beta$
Outline

1. Bag A and Bag B

2. Possible Decision Rules
   - Decision Rule #1
   - Decision Rule #2
   - Decision Rule #3

3. $\alpha$ vs. $\beta$

4. Assignment
If we modify the decision rule to decrease $\alpha$, we will increase $\beta$, and

If we modify the decision rule to decrease $\beta$, we will increase $\alpha$. 
α VS. β

Bag A

Bag B

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Bag A

Bag B

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Bag A

Bag B
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4. Assignment
Assignment

Homework

- Read Section 1.4 - 1.4.2, pages 16 - 28.
- Let’s Do It! 1.7, 1.8.