

Sampling Distributions

Sections 15.1, 15.2, 15.3

Lecture 26

Robb T. Koether

Hampden-Sydney College

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Outline

- 1 Parameters and Statistics
- 2 The Law of Large Numbers
- 3 Sampling Distributions
- 4 Assignment

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- 1 Parameters and Statistics
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Parameters and Statistics

Definition (Parameter)

A **parameter** is a number that describes some characteristic of a population.

Definition (Statistic)

A **statistic** is a number that describes some characteristic of a sample.

- The purpose of a statistic is to provide an estimate for a parameter.

Example

Example (Parameters and Statistics)

- According to the HSC Fact Book (online), the average GPA of all sophomores this year is 2.66.
- Suppose we survey a sample of 25 sophomores and find an average GPA of 2.98.
- Which number is a statistic and which is a parameter?

Parameters and Statistics

- For a given study,
 - The population mean is a parameter.
 - A sample mean is a statistic.
- A statistic is a random variable.

Parameters and Statistics

- For a given study,
 - The population mean is a parameter.
 - A sample mean is a statistic.
- A statistic is a random variable. How so?

Parameters and Statistics

- The population mean is denoted by μ , a parameter.
- A sample mean is denoted by \bar{x} , a statistic.
- \bar{x} is an **estimator** for μ .

Parameters and Statistics

- The population standard deviation is denoted by σ , a parameter.
- A sample standard deviation is denoted by s , a statistic.
- s is an **estimator** for σ .

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The Law of Large Numbers

The Law of Large Numbers

For any variable of a population, let μ be the mean of that variable. As sample sizes get larger and larger, the values of \bar{x} tends to get closer and closer to μ .

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Sampling Distributions

Definition (Sampling Distribution)

The **sampling distribution** of a statistic is the probability distribution of the values of that statistic over all possible samples of the same size.

Example

Example (Discrete Sampling Distribution)

- A population consists of 4 people A , B , C , and D .
- Their ages are 10, 12, 14, and 16, respectively.
- Let the sample size be 2 and find the sampling distribution of the sample mean.

Example

Example (Discrete Sampling Distribution)

- The possible samples (without replacement) are

Sample	Mean	Sample	Mean
$\{A, B\}$		$\{B, C\}$	
$\{A, C\}$		$\{B, D\}$	
$\{A, D\}$		$\{C, D\}$	

Example

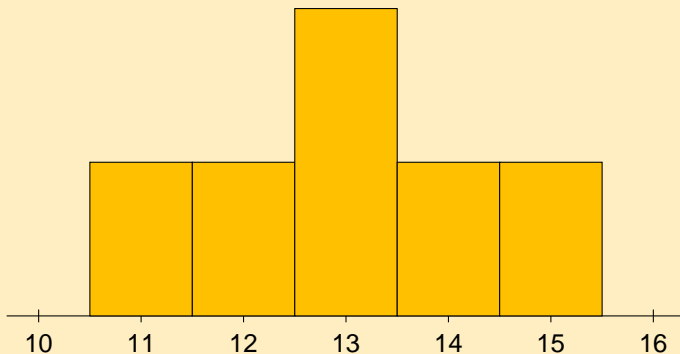
Example (Discrete Sampling Distribution)

- The possible samples (without replacement) are

Sample	Mean	Sample	Mean
$\{A, B\}$	11	$\{B, C\}$	13
$\{A, C\}$	12	$\{B, D\}$	14
$\{A, D\}$	13	$\{C, D\}$	15

Example

Discrete Sampling Distribution



Example

Example (Discrete Sampling Distribution)

Let x be the age of a randomly selected person and let \bar{x} be the average age of 2 randomly selected people.

- What is the probability that $x = 13$?

Example

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Example

Example (Discrete Sampling Distribution)

- Now suppose a population consists of 7 people $A, B, C, D, E, F,$ and G .
- Their ages are 10, 11, 12, 13, 14, 15, and 16, respectively.
- Let the sample size be 2 and find the sampling distribution of the sample mean.

Example

Example (Discrete Sampling Distribution)

- The possible samples (without replacement) are

Sample	Mean	Sample	Mean

Example

Example (Discrete Sampling Distribution)

- The possible samples (without replacement) are

Sample	Mean	Sample	Mean
{A, B}		{C, D}	
{A, C}		{C, E}	
{A, D}		{C, F}	
{A, E}		{C, G}	
{A, F}		{D, E}	
{A, G}		{D, F}	
{B, C}		{D, G}	
{B, D}		{E, F}	
{B, E}		{E, G}	
{B, F}		{F, G}	
{B, G}			

Example

Example (Discrete Sampling Distribution)

- The possible samples (without replacement) are

Sample	Mean	Sample	Mean
{A, B}	10.5	{C, D}	12.5
{A, C}	11.0	{C, E}	13.0
{A, D}	11.5	{C, F}	13.5
{A, E}	12.0	{C, G}	14.0
{A, F}	12.5	{D, E}	13.5
{A, G}	13.0	{D, F}	14.0
{B, C}	11.5	{D, G}	14.5
{B, D}	12.0	{E, F}	14.5
{B, E}	12.5	{E, G}	15.0
{B, F}	13.0	{F, G}	15.5
{B, G}	13.5		

Example

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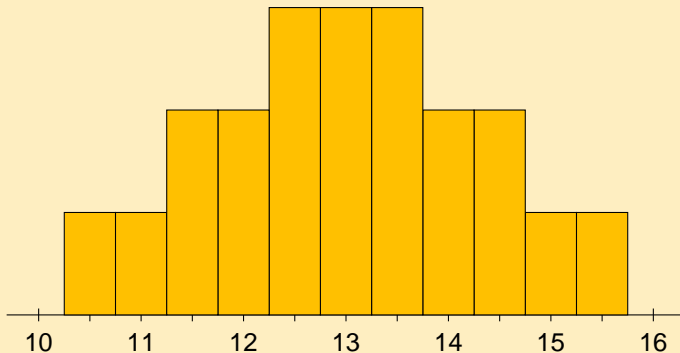
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Assignment

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- Read Sections 15.1, 15.2, 15.3.
- Apply Your Knowledge: 1, 2, 5, 6.
- Check Your Skills: 17, 18, 19.
- Exercises 25, 26, 27.