Tests of Significance
Sections 17.1, 17.2
Lecture 32

Robb T. Koether

Hampden-Sydney College

Thu, Mar 17, 2016

Outline

Outline

The Reasoning

- Suppose I told you that I had a “fair” coin, but when I tossed it 10 times, I got 6 heads and 4 tails?
- Would you doubt my claim?

The Reasoning

- Suppose I told you that I had a “fair” coin, but when I tossed it 10 times, I got 6 heads and 4 tails?
- Would you doubt my claim?
- What if I tossed it 25 times and got 15 heads and 10 tails?

The Reasoning

- Suppose I told you that I had a “fair” coin, but when I tossed it 10 times, I got 6 heads and 4 tails?
- Would you doubt my claim?
- What if I tossed it 25 times and got 15 heads and 10 tails?
- What if I tossed it 100 times and got 60 heads and 40 tails?

The Reasoning

- Suppose I told you that I had a “fair” coin, but when I tossed it 10 times, I got 6 heads and 4 tails?
- Would you doubt my claim?
- What if I tossed it 25 times and got 15 heads and 10 tails?
- What if I tossed it 100 times and got 60 heads and 40 tails?
- What if I tossed it 1000 times and got 520 heads and 480 tails?

The Reasoning

- The approach that we take is to say:

  IF the claim is true, THEN we expect to see . . .

The Reasoning

- The approach that we take is to say:

  IF the claim is true, THEN we expect to see . . .

- In other words, we entertain a hypothesis.

The Reasoning

- The approach that we take is to say:

  IF the claim is true, THEN we expect to see . . .
- In other words, we entertain a hypothesis.
- If we see what we would expect to see, then no problem.

The Reasoning

- The approach that we take is to say:

  IF the claim is true, THEN we expect to see . . .
- In other words, we entertain a hypothesis.
- If we see what we would expect to see, then no problem.
- But if not, then we have reason to reject the claim.

The Reasoning

- It so happens that the standard deviation for the number of heads out of n tosses is $\frac{\sqrt{n}}{2}$.
- With that fact in mind, evaluate each of the preceding scenarios.

Outline

The Hypotheses

- To test a hypothesis (a claim), we set up two opposing hypotheses.

The Hypotheses

- To test a hypothesis (a claim), we set up two opposing hypotheses.
- The null hypothesis is the claim that gets the benefit of the doubt.

The Hypotheses

- To test a hypothesis (a claim), we set up two opposing hypotheses.
- The null hypothesis is the claim that gets the benefit of the doubt.
- The alternative hypothesis is the opposing claim and it bears the burden of proof.

The Hypotheses

- To test a hypothesis (a claim), we set up two opposing hypotheses.
- The null hypothesis is the claim that gets the benefit of the doubt.
- The alternative hypothesis is the opposing claim and it bears the burden of proof.
- The null hypothesis is denoted by the symbol H_0 and the alternative hypothesis is denoted by H_a.

The Hypotheses

- The null hypothesis always states a specific value for a specific population parameter.
- The alternative hypothesis always contradicts the null hypothesis by stating that the parameter is
  - Not equal to that value, or
  - Greater than that value, or
  - Less than that value,

  depending on the circumstances.

Outline

Example

Example (Coin Tossing)

- In the coin-tossing example, state the null hypothesis.
- What is the alternative hypothesis?
- What constitutes sufficient “proof” to support the alternative hypothesis?

Example

Example (Die Rolling)

- A fair die will produce the numbers 1, 2, 3, 4, 5, and 6 with equal frequencies, in the long run.
- Thus, the “population” of all possible die rolls has a mean of 3.5 and a standard deviation of 1.7078.
- If I claim that the die is fair, then

Example

Example (Die Rolling)

- A fair die will produce the numbers 1, 2, 3, 4, 5, and 6 with equal frequencies, in the long run.
- Thus, the “population” of all possible die rolls has a mean of 3.5 and a standard deviation of 1.7078.
- If I claim that the die is fair, then
  - What should be the null hypothesis?

Example

Example (Die Rolling)

- A fair die will produce the numbers 1, 2, 3, 4, 5, and 6 with equal frequencies, in the long run.
- Thus, the “population” of all possible die rolls has a mean of 3.5 and a standard deviation of 1.7078.
- If I claim that the die is fair, then
  - What should be the null hypothesis?
  - What should be the alternative hypothesis?

Example

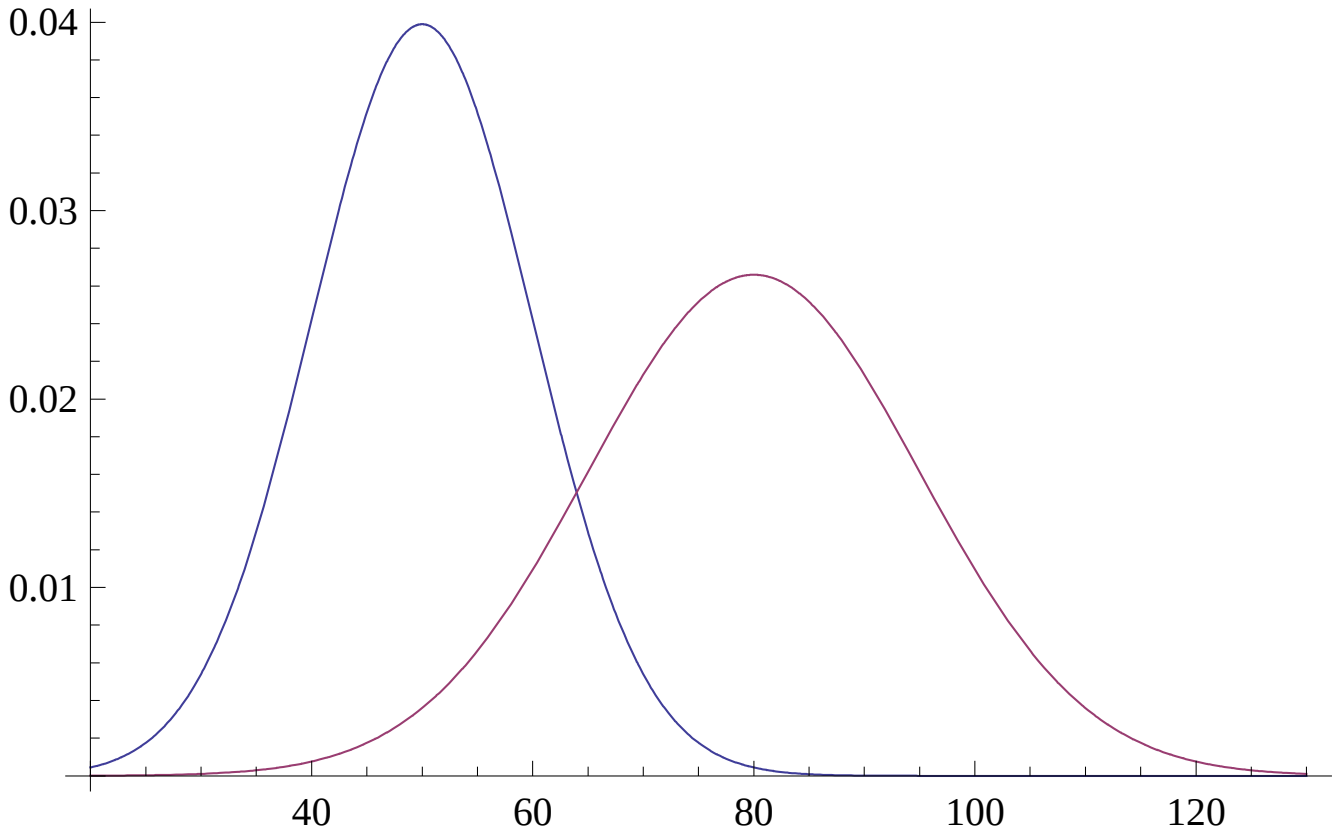
Example (Die Rolling)

- A fair die will produce the numbers 1, 2, 3, 4, 5, and 6 with equal frequencies, in the long run.
- Thus, the "population" of all possible die rolls has a mean of 3.5 and a standard deviation of 1.7078.
- If I claim that the die is fair, then
  - What should be the null hypothesis?
  - What should be the alternative hypothesis?
- Run the computer program `DieTest.exe` to experiment.

Drug User vs. Non-drug User

- A subject is either a drug user or a non-drug user.
- To determine which, we measure the level of Substance "X" in his blood.
- Among non-drug users, the level of X has the distribution $N(50, 10)$.
- Among drug users, the level of X has distribution $N(80, 15)$.

Drug User vs. Non-drug User



Drug User vs. Non-drug User

- We take a blood sample and measure the amount of X.
- What is the null hypothesis?
- What is the alternative hypothesis?

Drug User vs. Non-drug User

- Decision Rule: If the level of X is more than 70 (2 standard deviations), then we will decide that the subject is a drug user.

Drug User vs. Non-drug User

- If he is not a drug user, what is the probability that the test will conclude that he is a drug user?
- That would be a false positive.

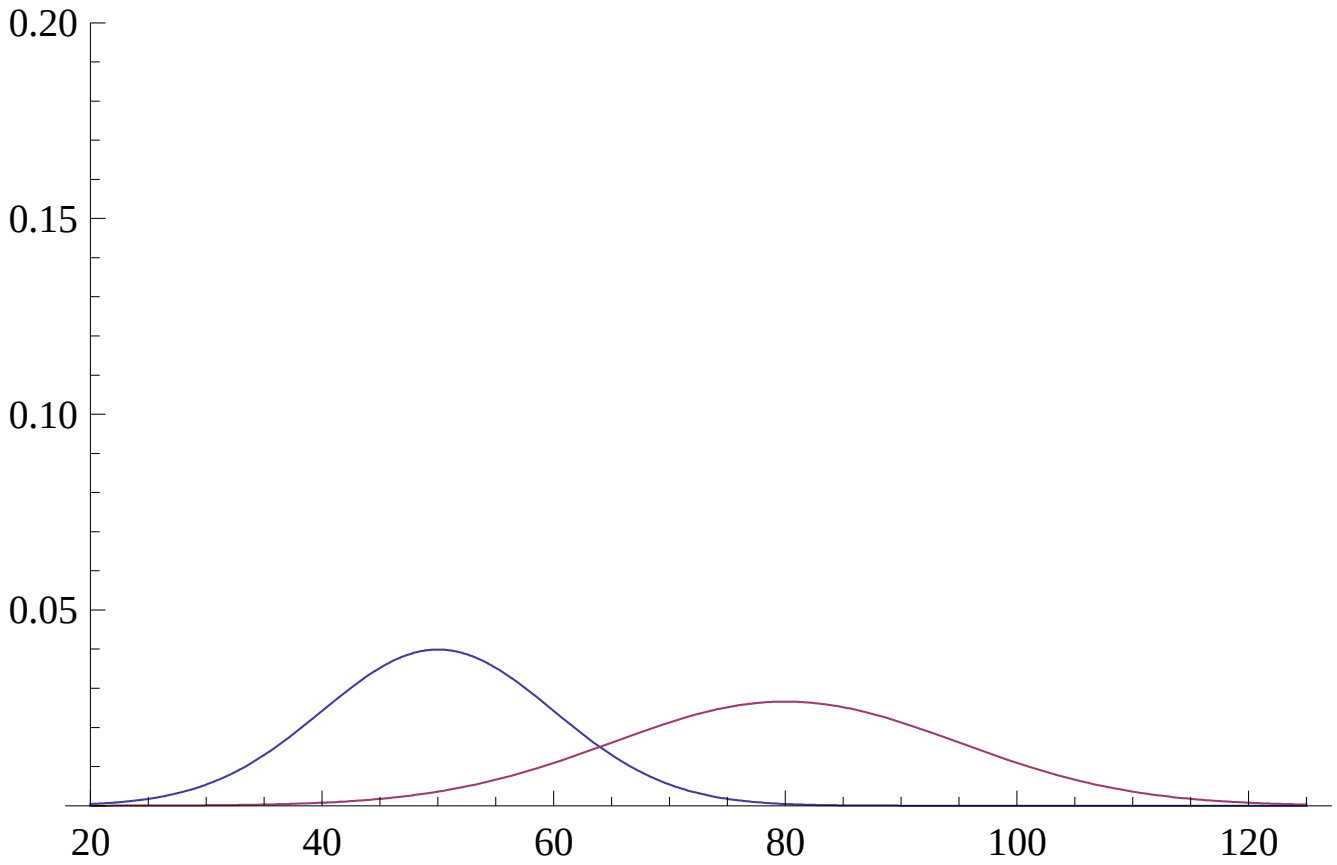
Drug User vs. Non-drug User

- If he is not a drug user, what is the probability that the test will conclude that he is a drug user?
- That would be a false positive.
- If he is a drug user, what is the probability that the test will conclude that he is not a drug user?
- That would be a false negative.

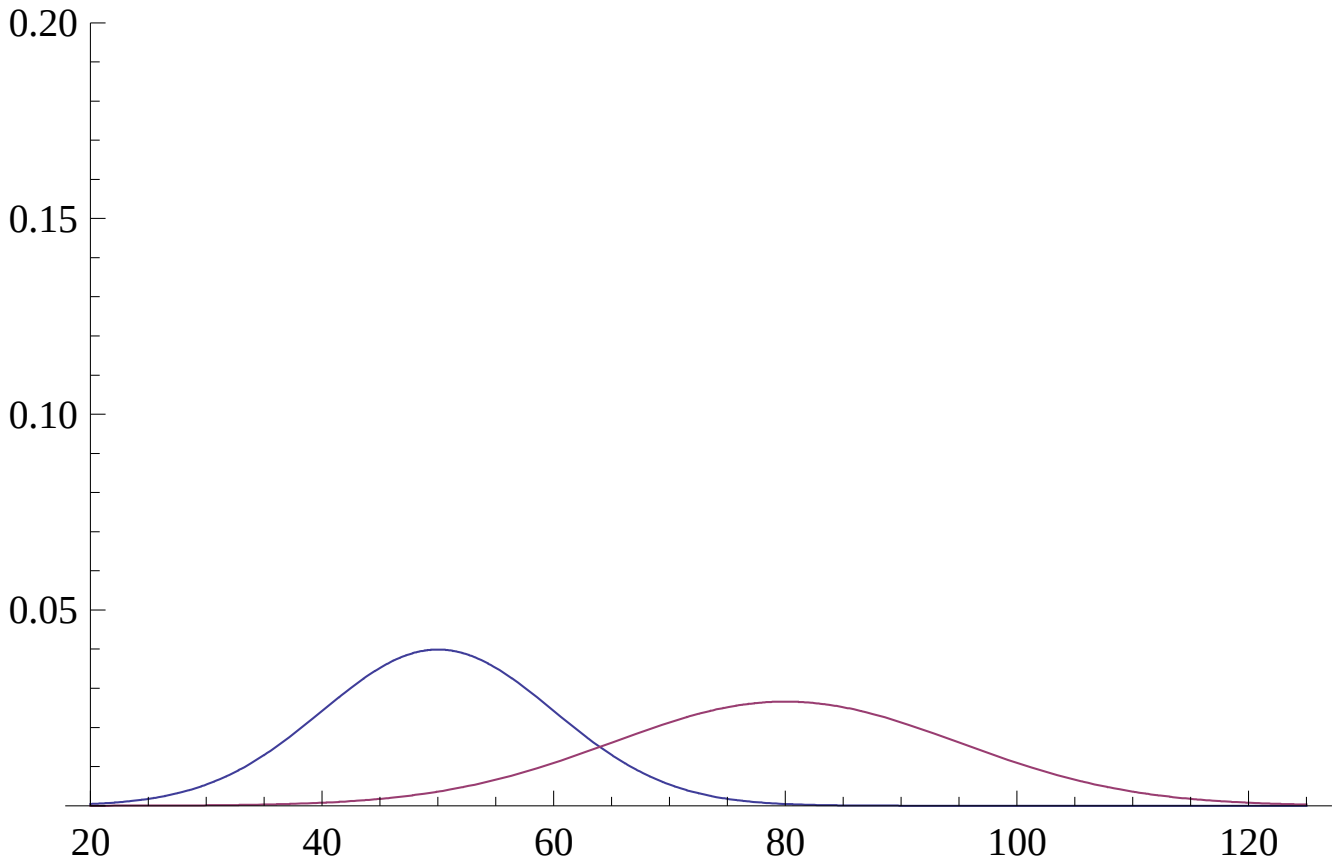
Drug User vs. Non-drug User

- If he is not a drug user, what is the probability that the test will conclude that he is a drug user?
- That would be a false positive.
- If he is a drug user, what is the probability that the test will conclude that he is not a drug user?
- That would be a false negative.
- Which type of error is more serious?

Drug User vs. Non-drug User

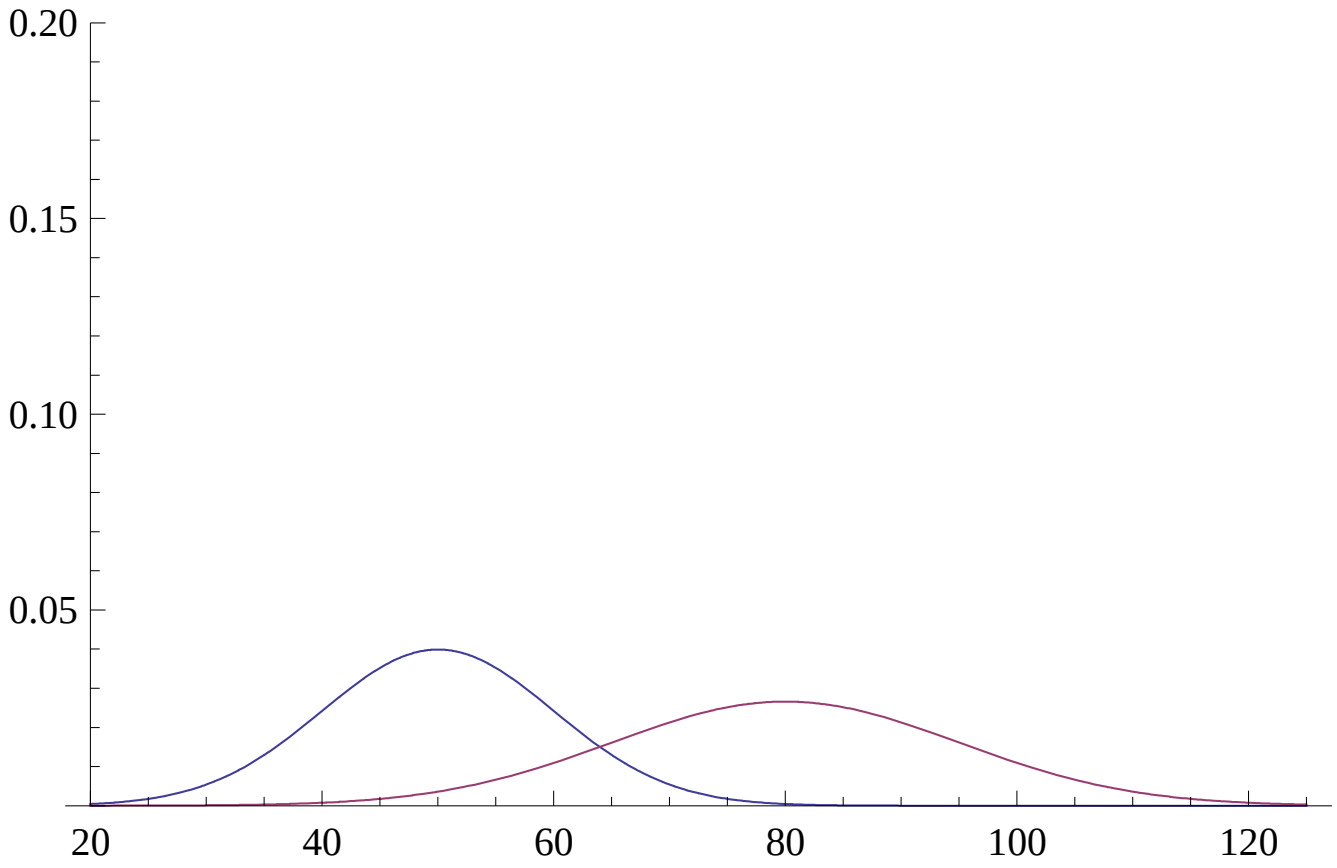


Drug User vs. Non-drug User



normalcdf(60,E99,50,10) = 0.1587

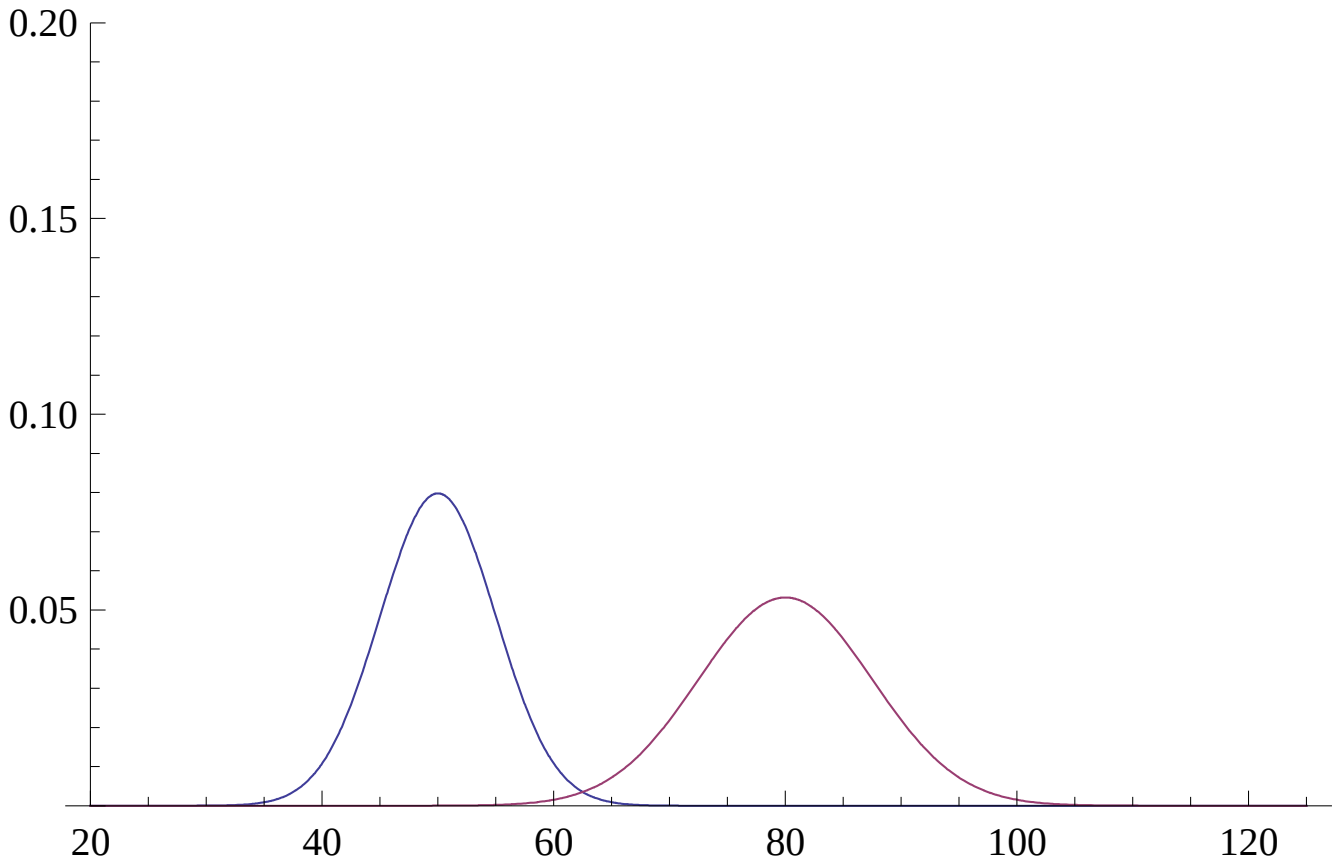
Drug User vs. Non-drug User



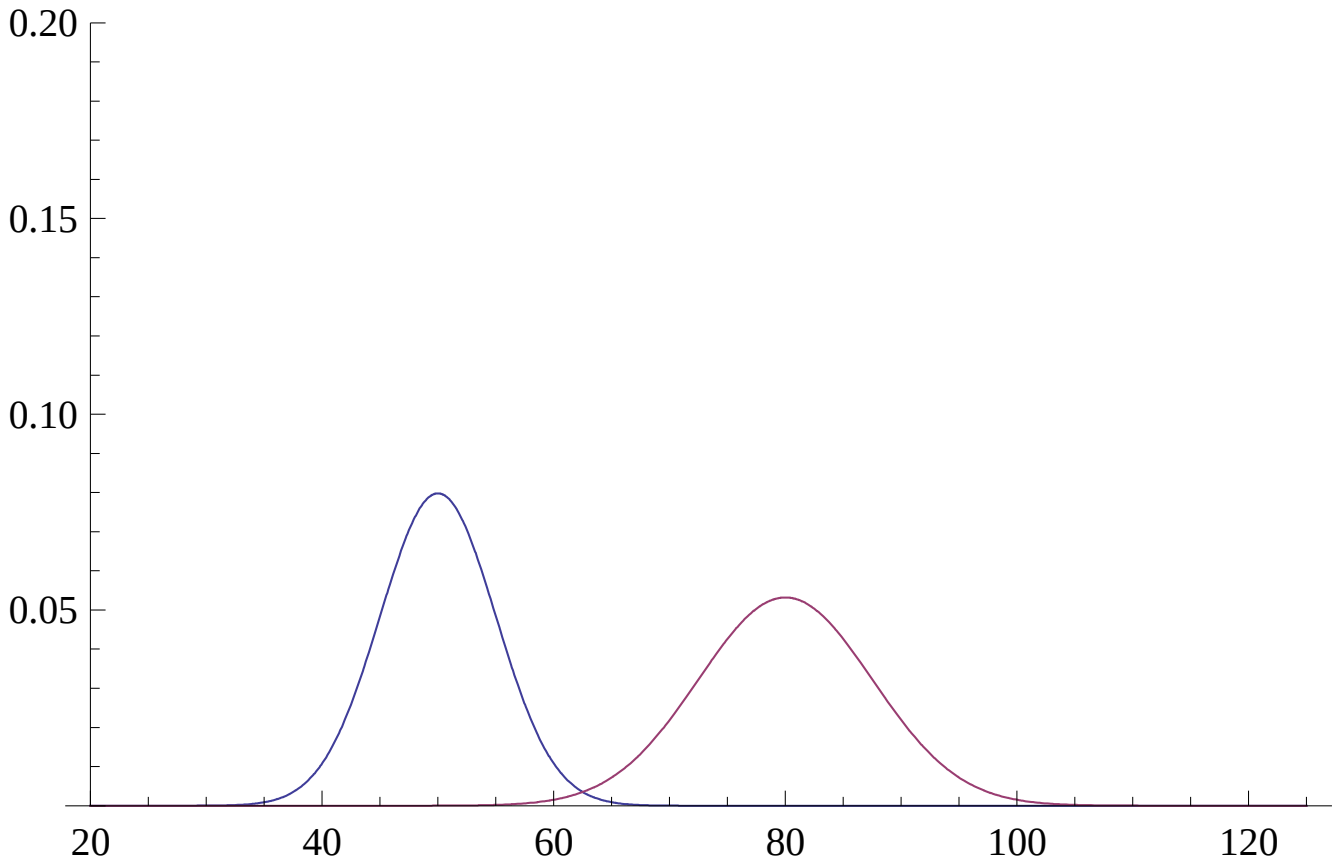
normalcdf(60,E99,50,10) = 0.1587

normalcdf(-E99,60,80,15) = 0.0912

Drug User vs. Non-drug User

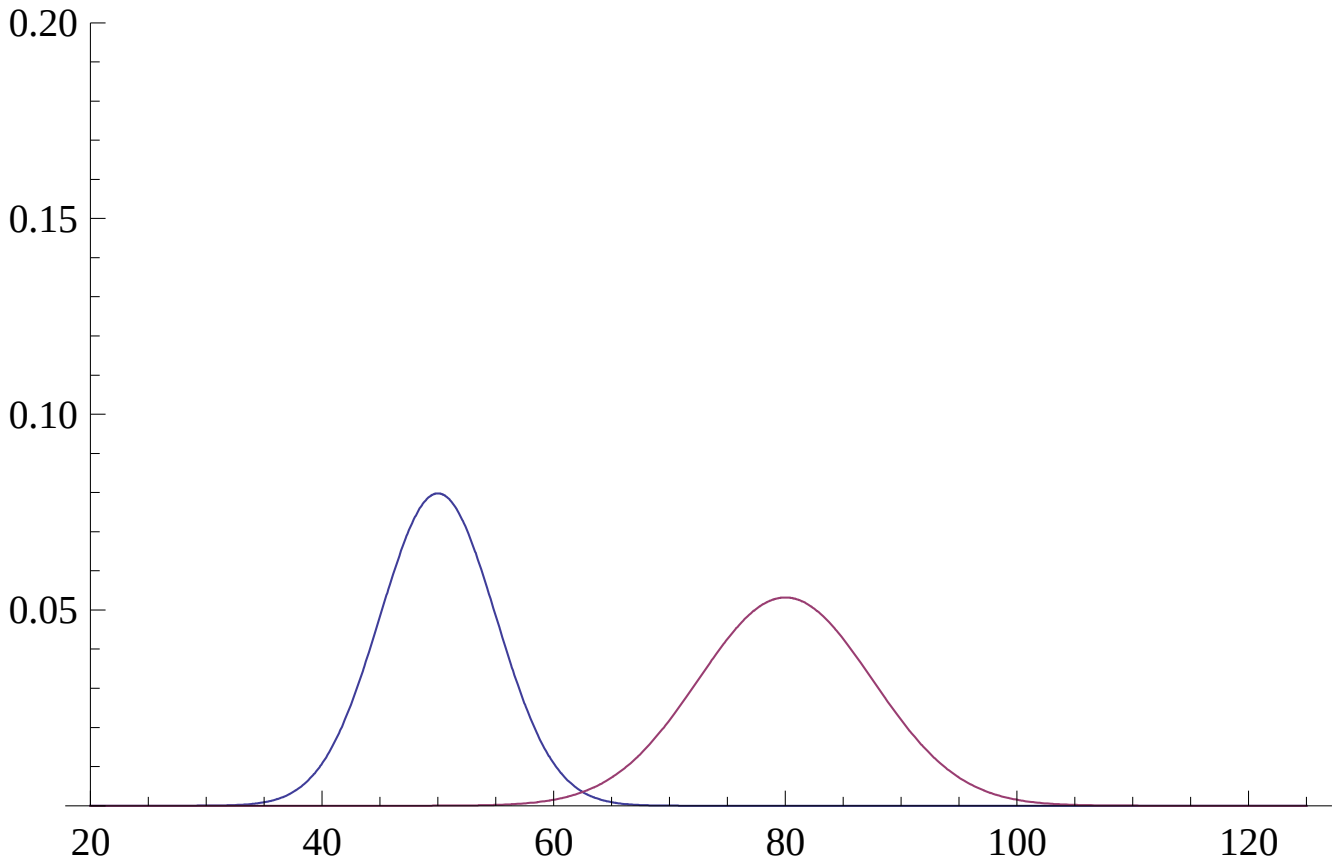


Drug User vs. Non-drug User



normalcdf(60,E99,50,5) = 0.0228

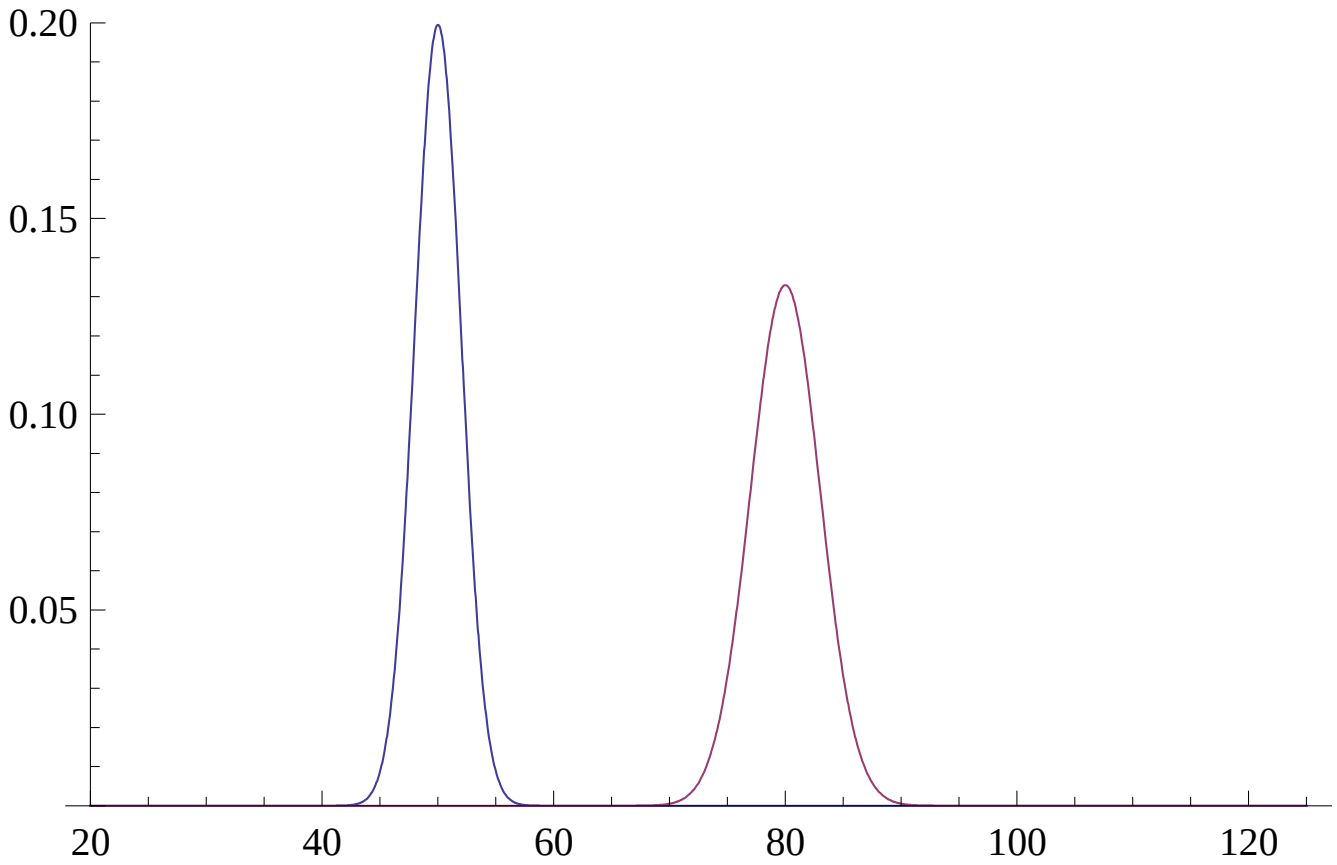
Drug User vs. Non-drug User



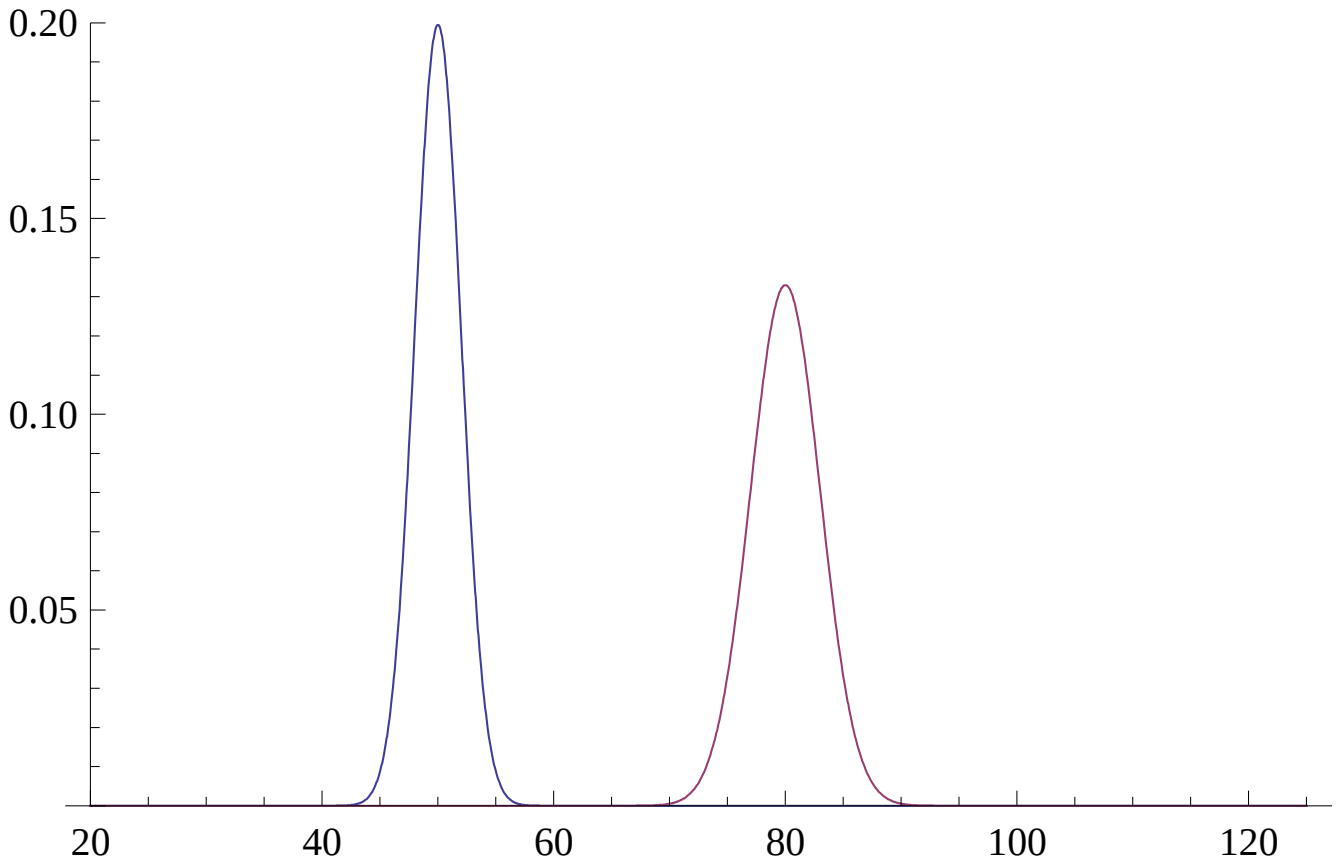
normalcdf(60,E99,50,5) = 0.0228

normalcdf(−E99,60,80,7.5) = 0.0038

Drug User vs. Non-drug User

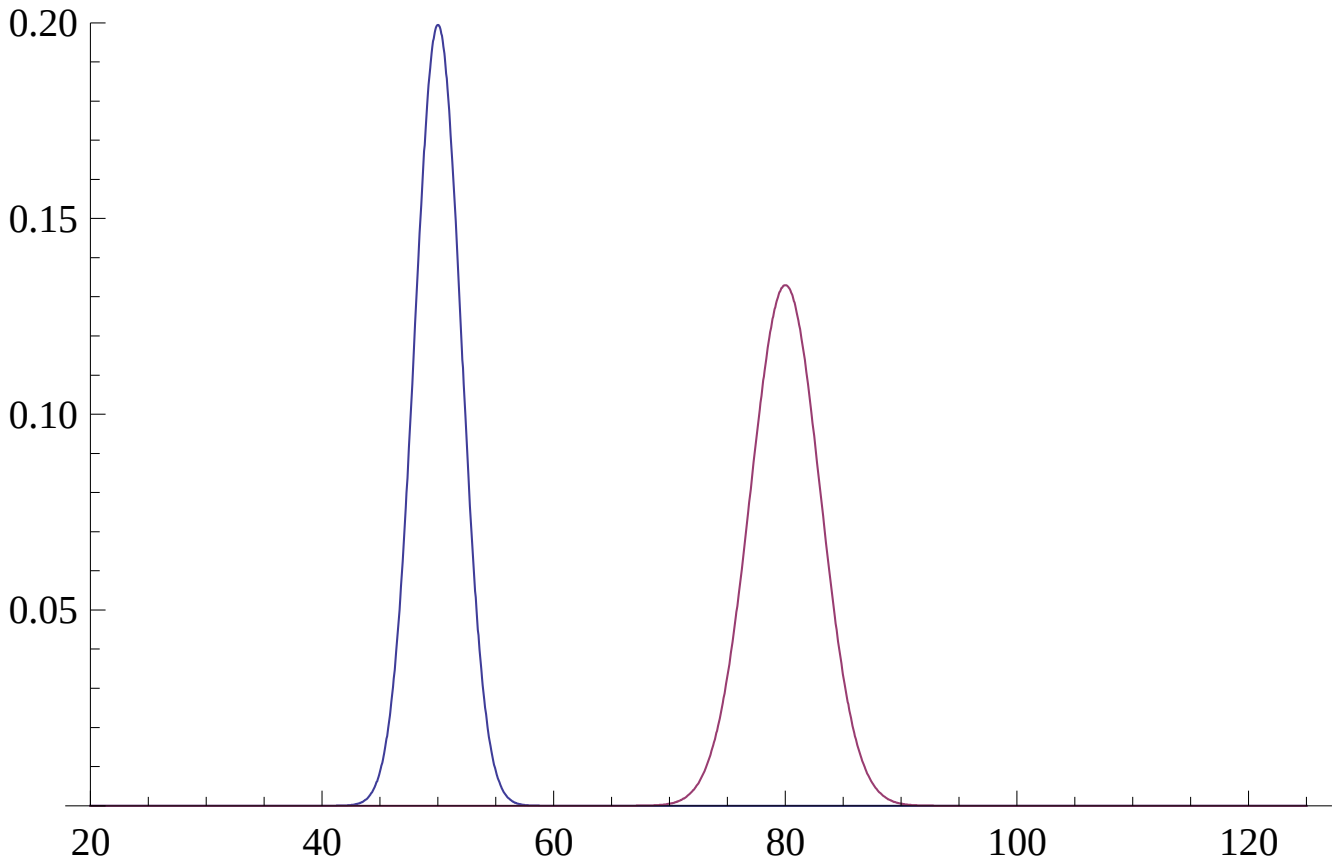


Drug User vs. Non-drug User



normalcdf(60,E99,50,2) = 0.0000

Drug User vs. Non-drug User



normalcdf(60,E99,50,2) = 0.0000

normalcdf(-E99,60,80,3) = 0.0000

Outline

Assignment

Assignment

- Read Sections 17.1, 17.2.
- Apply Your Knowledge: 1, 3, 5, 6, 7.