Matched Pairs
Sections 20.6, 20.7
Lecture 37

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Outline

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1. When to Use t
2. When to Use z
3. Independent vs. Dependent Samples
4. Matched Pairs
5. Example
6. Assignment

- Use the t statistic if we are using s in place of σ and
  - The sample size is small ($n \leq 15$) and the data are close to normal, or

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  - The sample size is larger ($15 < n < 40$) and the data are not strongly skewed nor do they contain any outliers, or
  - The sample size is large ($n \ge 40$).

Outline

- Use the z statistic if we know σ and
    - The data are close to normal, or

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Dependent Samples

Definition (Bivariate Data, Matched Pairs)

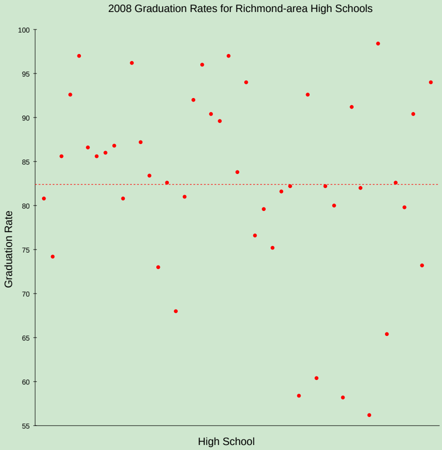
Bivariate data are data in which each datum is a pair of observations. These are also called paired data. Typically the two values are called x_1 and x_2. The sample of x_1 values and the sample of x_2 values are called matched pairs, or paired samples, or dependent samples.

Dependent Samples

- Matched pairs are often obtained in “before” and “after” studies.
- By comparing the mean before treatment to the mean after treatment, we can determine whether the treatment had an effect.
- To make direct comparisons of the two samples, they must be measuring the same sort of thing.
- Clearly, paired samples must be of the same size.

Dependent Samples

Example (High-School Graduation Rates)



Dependent Samples

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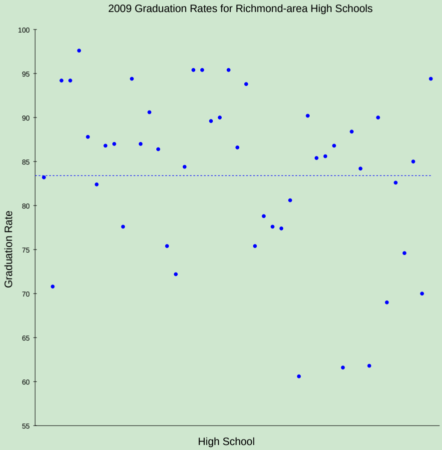

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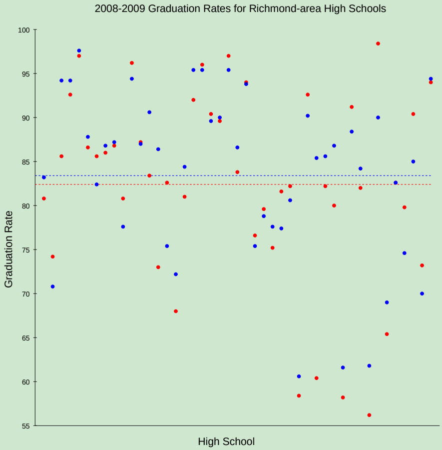

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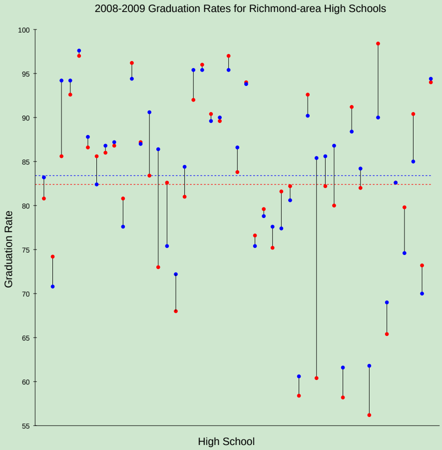

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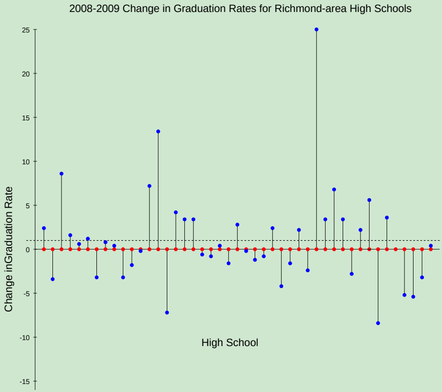

Example (High-School Graduation Rates)

- Was there an overall improvement in the graduation rate?
- That is, is the average difference greater than 0?

Independent Samples

- On the other hand, with independent samples, we simply take one sample from one population and another sample from another population.
- There is no logical way to pair the data.
- Furthermore, the independent samples could be of different sizes.
- We will study independent samples in Chapter 21.

Outline

Matched Pairs

- Let the pairs be denoted (x_1, x_2).
- Let $x = x_2 - x_1$.
- We will study the case where the population of differences has a normal distribution.
- As usual, let μ and σ denote the mean and standard deviation of the population of differences.

Hypothesis Testing for Matched Pairs

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Hypothesis Testing for Matched Pairs

- The only null hypothesis for μ that we will consider with matched pairs is

$$H_0 : \mu = 0.$$

- We will consider any of the three alternatives

$$H_1 : \mu < 0.$$
$$H_1 : \mu > 0.$$
$$H_1 : \mu \neq 0$$

Hypothesis Testing for Matched Pairs

Example (Hypothesis Testing for Matched Pairs)

- If the population is normal or approximately normal, then the test statistic is
$$t = \frac{\overline{x} - 0}{s/\sqrt{n}}.$$
- If the sample size is large, then we can use either t or z.

Outline

Example

Example (Hypothesis Testing for Matched Pairs)

- Suppose that a group of 10 students take a math placement test.
- Let the variable x_1 represent their scores on that test.
- Then they are given an Algebra refresher course and they retake the placement test.
- Let the variable x_2 represent their scores on the retest.

Example

Example (Hypothesis Testing for Matched Pairs)

- The following table shows the results

Student	1st Score (x_1)	2nd Score (x_2)	Difference (x)
1	83	81	
2	62	63	
3	80	76	
4	73	80	
5	68	78	
6	67	71	
7	68	69	
8	69	78	
9	80	88	
10	83	79	

Example

Example (Hypothesis Testing for Matched Pairs)

- The following table shows the results

Student	1st Score (x_1)	2nd Score (x_2)	Difference (x)
1	83	81	-2
2	62	63	1
3	80	76	-4
4	73	80	7
5	68	78	10
6	67	71	4
7	68	69	1
8	69	78	9
9	80	88	8
10	83	79	-4

Example

Example (Hypothesis Testing for Matched Pairs)

- Test the hypothesis, at the 10% level, that the refresher course improved their grades on the placement test.

Example

Example (Hypothesis Testing for Matched Pairs)

(1) Let x_1 be the first test score, let x_2 be the second test score, and let $x = x_2 - x_1$.
Then the hypotheses are

$$H_0 : \mu = 0.$$
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$$H_1 : \mu > 0$$

(2) $\alpha = 0.10$.

(3) Let $t = \dfrac{\overline{x} - 0}{s/\sqrt{n}}$.

Example

Example (Hypothesis Testing for Matched Pairs)

(4) Compute the value of the test statistic.

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- We find that $\overline{x} = 3$ and $s = 5.354$.

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- Evaluate the difference $\mathtt{L}_2 - \mathtt{L}_1$ and store it in $\mathtt{L}_3$.
- Use `1-Var Stats` $\mathtt{L}_3$ to get $\overline{x}$ and s.
- We find that $\overline{x} = 3$ and $s = 5.354$.
- Then

$$t = \frac{3}{5.354/\sqrt{10}} = \frac{3}{1.693} = 1.772.$$

Example

Example (Hypothesis Testing for Matched Pairs)

(5) p-value $=$ `tcdf(1.772,E99,9)` $=$ 0.0551.

Example

Example (Hypothesis Testing for Matched Pairs)

(5) p-value $=$ `tcdf(1.772,E99,9)` $=$ 0.0551.

(6) Reject H_0 and conclude that the students' scores on the placement test are higher after taking the Algebra refresher course.

Outline

Assignment

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- Read Sections 20.6, 20.7.
- Apply Your Knowledge: 11, 12, 13.
- Check Your Skills: 25, 26.
- Exercises 35, 39, 42, 48, 51.