The Goodness-of-Fit Test
Section 25.9
Lecture 47

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Outline

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Goodness-of-Fit

- For a goodness-of-fit test, we have *one* categorical variable and a hypothetical distribution for that variable.
- is the hypothetical distribution the correct distribution?
- For example, a fair die should land 1, 2, 3, 4, 5, and 6 with equal frequencies, in the long run.

1	2	3	4	5	6	Total
12	9	7	13	10	9	60

- Is the die fair?

Goodness-of-Fit

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Goodness-of-Fit

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- The statistic is the same as before:

$$\chi^2 = \sum_{\text{all cells}} \frac{(O-E)^2}{E}.$$

Goodness-of-Fit

- Calculate

$$\begin{aligned}
\chi^2 &= \sum_{\text{all cells}} \frac{(O-E)^2}{E} \\
&= \frac{(12-10)^2}{10} + \frac{(9-10)^2}{10} + \frac{(7-10)^2}{10} \\
&\quad + \frac{(13-10)^2}{10} + \frac{(10-10)^2}{10} + \frac{(9-10)^2}{10} \\
&= 0.4 + 0.1 + 0.9 + 0.9 + 0.0 + 0.1 \\
&= 2.4.
\end{aligned}$$

Goodness-of-Fit

- Is 2.4 an extreme value for χ^2, or is it an "ordinary" value?
- That depends on the number of degrees of freedom.
- In the Goodness-of-Fit Test,

$$\text{degrees of freedom} = \text{No. of cells} - 1.$$

- In this example, we have 5 degrees of freedom.
- Therefore, the expected value of χ^2 is 5 and the standard deviation is $\sqrt{10} \approx 3.2$.
- The value 2.4 is actually *smaller* than expected.

Outline

Assignment

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- Read Sections 25.9.
- Apply Your Knowledge: 15, 16, 17, 18.