

Chi-Square vs. z

Section 25.9

Lecture 49

Robb T. Koether

Hampden-Sydney College

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- 1 χ^2 Versus z
 - Goodness-of-fit Test, 2 Categories
 - 2×2 Table
- 2 Assignment

Example

- There are two situations where the χ^2 test has 1 degree of freedom.
 - A goodness-of-fit test with only 2 categories.
 - A two-way table with 2 rows and 2 columns.

Example

- There are two situations where the χ^2 test has 1 degree of freedom.
 - A goodness-of-fit test with only 2 categories.
 - A two-way table with 2 rows and 2 columns.
- In these cases, the test could be performed as a z-test.
 - Goodness-of-fit test \Rightarrow Test of one proportion.
 - 2×2 table \Rightarrow Comparing two proportions.

Outline

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Goodness-of-fit Test, 2 Categories

- To test a coin for fairness, we toss it 1000 times and get 467 heads and 533 tails.

Goodness-of-fit Test, 2 Categories

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- Using the z-test:

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 - The sample proportion is $\hat{p} = 0.467$.

Goodness-of-fit Test, 2 Categories

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- Using the z-test:
 - The sample proportion is $\hat{p} = 0.467$.
 - The test statistic is

$$\begin{aligned}z &= \frac{0.467 - 0.500}{\sqrt{\frac{(0.5)(0.5)}{1000}}} \\ &= \frac{-0.033}{0.0158} \\ &= -2.087.\end{aligned}$$

Goodness-of-fit Test, 2 Categories

- Using the goodness-of-fit test:

Goodness-of-fit Test, 2 Categories

- Using the goodness-of-fit test:
 - The observed and expected counts:

	Heads	Tails	Total
Obs	467	533	1000
Exp	500	500	1000

Goodness-of-fit Test, 2 Categories

- Using the goodness-of-fit test:
 - The observed and expected counts:

	Heads	Tails	Total
Obs	467	533	1000
Exp	500	500	1000

- The test statistic is

$$\begin{aligned}\chi^2 &= \frac{(467-500)^2}{500} + \frac{(533-500)^2}{500} \\ &= 2.178 + 2.178 \\ &= 4.356.\end{aligned}$$

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2 × 2 Table

- To test two coins to see whether they land heads equally often, we toss each coin 1000 times.

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- Using the z-test:
 - The sample proportions are $\hat{p}_1 = 0.600$ and $\hat{p}_2 = 0.550$.
 - The pooled estimate is $\hat{p} = \frac{600+550}{1000+1000} = \frac{1150}{2000} = 0.575$.

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 - The sample proportions are $\hat{p}_1 = 0.600$ and $\hat{p}_2 = 0.550$.
 - The pooled estimate is $\hat{p} = \frac{600+550}{1000+1000} = \frac{1150}{2000} = 0.575$.
 - The test statistic is

$$\begin{aligned} z &= \frac{0.600 - 0.550}{\sqrt{(0.575)(1 - 0.575) \left(\frac{1}{1000} + \frac{1}{1000}\right)}} \\ &= \frac{0.050}{0.022107} \\ &= 2.26165. \end{aligned}$$

2×2 Table

- Using the goodness-of-fit test:

2×2 Table

- Using the goodness-of-fit test:
 - The 2×2 table is

	Heads	Tails	Total
Coin 1	600	400	1000
Coin 2	550	450	1000
	1150	850	2000

2×2 Table

- Using the goodness-of-fit test:
 - The 2×2 table is

	Heads	Tails	Total
Coin 1	600 575	400 425	1000
Coin 2	550 575	450 425	1000
	1150	850	2000

2 × 2 Table

- Using the goodness-of-fit test:
 - The 2 × 2 table is

	Heads	Tails	Total
Coin 1	600	400	1000
Coin 2	550	450	1000
	1150	850	2000

- The test statistic is

$$\begin{aligned}\chi^2 &= \frac{(600-575)^2}{575} + \frac{(400-425)^2}{425} + \frac{(550-575)^2}{575} + \frac{(450-425)^2}{425} \\ &= 1.0869 + 1.4706 + 1.0869 + 1.4706 \\ &= 5.1151.\end{aligned}$$

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- Read Sections 25.9.