Normal Distributions
Sections 3.3, 3.4, 3.5, 3.6
Lecture 9

Robb T. Koether

Hampden-Sydney College

Thu, Jan 28, 2016

Outline

Outline

The Normal Density Curve

Definition (The Normal Density Curve)

A normal density curve has a very specific shape.

- It is symmetric.
- It has a single, central peak.
- The curve drops steadily to the left and right of the peak.
- The curve extends forever in both directions.
- The "main part" of the curves lies between 3 standard deviations below the mean and 3 standard deviations above the mean.

Outline

Tossing a Coin

- Suppose a coin is tossed 10,000 times and the number of heads is counted.
- What is the distribution of the number of heads?

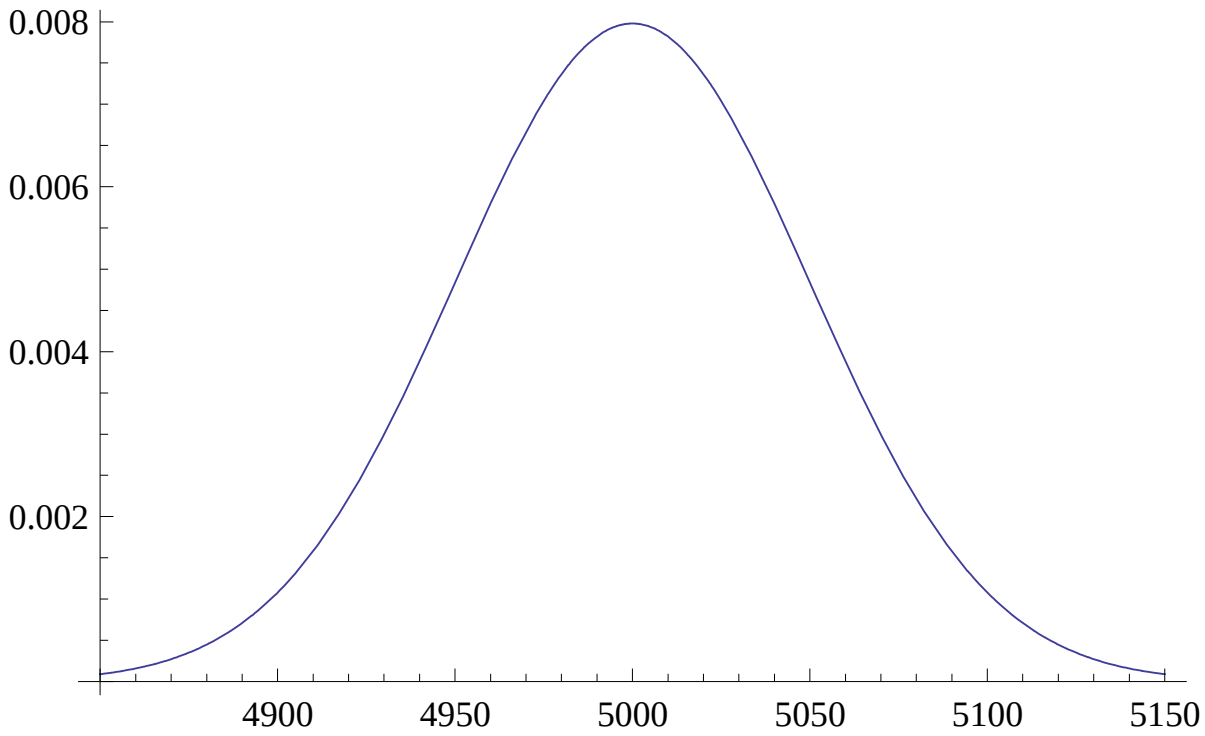
Tossing a Coin

- Suppose a coin is tossed 10,000 times and the number of heads is counted.
- What is the distribution of the number of heads?
- It is normal with mean $\mu = 5,000$ and $\sigma = 50$.

Tossing a Coin

- Suppose a coin is tossed 10,000 times and the number of heads is counted.
- What is the distribution of the number of heads?
- It is normal with mean $\mu = 5,000$ and $\sigma = 50$.
- Sketch the shape of that distribution, including the scale.

The Normal Density Curve



Rolling a Die

- Suppose a die is rolled 720 times and the number of sixes is counted.
- What is the distribution of the number of sixes?

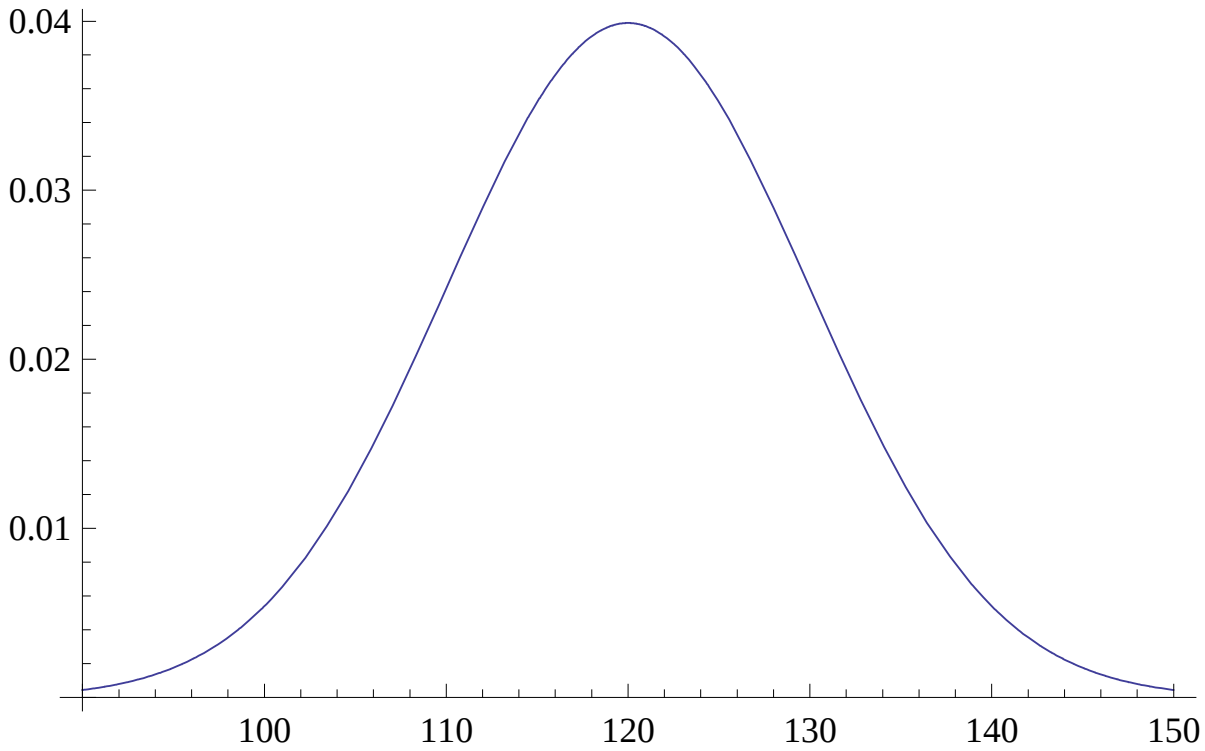
Rolling a Die

- Suppose a die is rolled 720 times and the number of sixes is counted.
- What is the distribution of the number of sixes?
- It is normal with mean $\mu = 120$ and $\sigma = 10$.

Rolling a Die

- Suppose a die is rolled 720 times and the number of sixes is counted.
- What is the distribution of the number of sixes?
- It is normal with mean $\mu = 120$ and $\sigma = 10$.
- Sketch the shape of that distribution, including the scale.

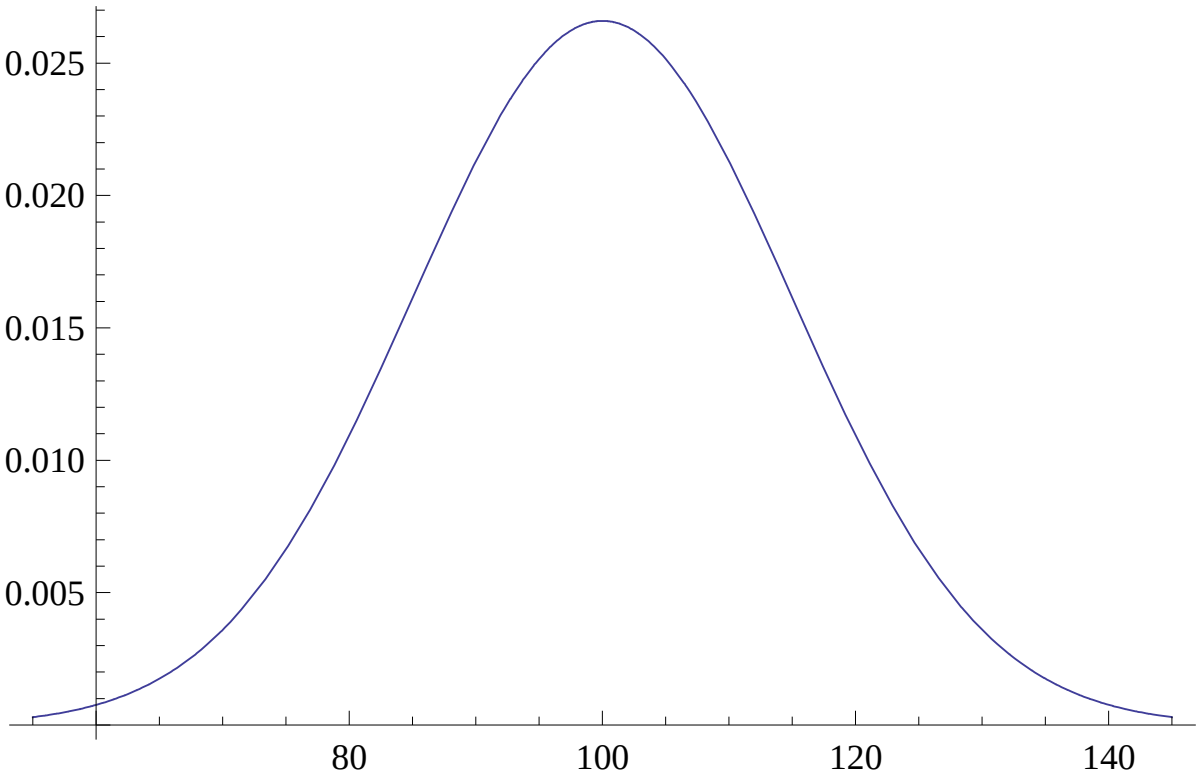
The Normal Density Curve



IQ Scores

- IQ scores have an approximately normal distribution with $\mu = 100$ and $\sigma = 15$.
- Sketch the shape of that distribution, including the scale.

The Normal Density Curve



Outline

The 68-95-99.7 Rule

The 68-95-99.7 Rule

The 68-95-99.7 Rule says that

- Approximately 68% of the observations fall within σ of μ. That is, between $\mu - \sigma$ and $\mu + \sigma$.
- Approximately 95% of the observations fall within 2σ of μ. That is, between $\mu - 2\sigma$ and $\mu + 2\sigma$.
- Approximately 99.7% of the observations fall within 3σ of μ. That is, between $\mu - 3\sigma$ and $\mu + 3\sigma$.

The 68-95-99.7 Rule

- Apply this rule to the coin-tossing, die-rolling, and IQ examples.

The 68-95-99.7 Rule

- What proportion of the observations lie
  - Between the μ and $\mu + \sigma$?
  - Between the μ and $\mu + 2\sigma$?
  - Between the $\mu + \sigma$ and $\mu + 2\sigma$?
  - Greater than $\mu + \sigma$?

Outline

z-Scores

Definition (z-Score)

If x is an observation from a distribution that has mean μ and standard deviation σ, then the standardized value, or z-score, of x is

$$z = \frac{x - \mu}{\sigma}.$$

- The z-score is a measure of the number of standard deviations the observation is above or below average.
- z-scores greater than 2 or less than -2 are rare. (How rare?)
- z-scores greater than 3 or less than -3 are very rare. (How rare?)

Outline

Assignment

Assignment

- Read Sections 3.3 - 3.6.
- Apply Your Knowledge: 5, 6, 7, 8, 9.
- Check Your Skills: 16, 17, 18, 21.
- Exercises: 26, 27.