In all hypothesis-testing problems, show all seven steps.

1. (10 pts) Suppose that 60% of the HSC student body is in favor of tearing down Bagby Hall and replacing it with a new Mathematics Center. If the student government takes a survey of 100 students, selected at random, what is the probability that fewer than half of them will support the action?

2. (15 pts) It is estimated that 80% of the adult residents of Idaho own guns. A researcher wishes to see whether the proportion of Virginia residents who own guns is lower than that, so he will test the hypothesis $H_0 : p = 0.80$ against the hypothesis $H_1 : p < 0.80$. He surveys 558 adult residents in Virginia, selected at random, and finds that 423 of them own guns. Carry out the test at the 5% level of significance.

3. (15 pts) The researcher of the previous problem decides to estimate the proportion of Virginia residents who own guns. Use the data from the previous problem to find a 95% confidence interval for this proportion.

4. (15 pts) The Lundberg Survey reported that the average price of gasoline was $\bar{x}_1 = \$2.20$ on Oct. 20 and $\bar{x}_2 = \$2.18$ on Nov. 3. Both surveys were based on $n = 7000$ gas stations. The sample standard deviations were not reported, but let us assume that the sample standard deviation was $s_1 = 0.13$ on Oct. 20 and $s_2 = 0.11$ on Nov. 3. Test the hypothesis that the true average price of gasoline was lower on Nov. 3 than it was on Oct. 20, i.e., $\mu_1 > \mu_2$. Use a significance level of 5%.

5. (15 pts) A group of researchers is considering introducing a new manufacturing process into a factory. They have two processes under consideration: Process 1 and Process 2. In a trial, Process 1 was used to produce 200 items and 18 of them were found to be defective. In another trial, Process 2 was used to produce 250 items and 14 of them were found to be defective. The researchers are interested in whether there is any difference in the rate of defective items between the two processes. Perform the test of the researchers’ hypothesis at the 1% level of significance.

6. (15 pts) A random sample of 10 sub sandwiches produced by a certain sub shop showed carbohydrate contents of 47, 61, 49, 47, 47, 63, 45, 48, 48, and 45 grams. Assume that the carbohydrate content of all sub sandwiches from this shop has a normal distribution. Test the hypothesis at the 5% level of significance that the average carbohydrate content of this sub shop’s sandwiches is greater than 47 grams.

7. (15 pts) Use the data of the previous problem to construct a 90% confidence interval to estimate the carbohydrate content of the sub sandwiches.