Closure Properties of Decidable and Recognizable Languages

Lecture 28

Problems 3.15 and 3.16

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Outline

1. Homework Review
2. Closure Properties of Decidable Languages
   - Intersection
   - Union
3. Closure Properties of Recognizable Languages
   - Intersection
   - Union
4. Assignment

Homework Review

Exercise 3.6, page 160.

In Theorem 3.21 we showed that a language is Turing-recognizable iff some enumerator enumerates it. Why didn't we use the following simpler algorithm for the forward direction of the proof? As before, $s_1, s_2, \ldots$ is a list of all strings in Σ^*.

$E =$ "Ignore the input.

1. Repeat the following for $i = 1, 2, 3, \ldots$
2. Run M on s_i.
3. If it accepts, print out s_i."

Homework Review

Exercise 3.7, page 160.

Explain why the following is not a description of a legitimate Turing machine.

M_{bad} = "The input is a polynomial p over variables $x_1, \ldots, x_n$.

1. Try all possible settings of $x_1, \ldots, x_n$ to integer values.
2. Evaluate p on all of these settings.
3. If any of these settings evaluates to 0, *accept*; otherwise, *reject*."

Closure Properties of Decidable Languages

Theorem (Closure Properties of Decidable Languages)

The class of decidable languages is closed under

- *Union*
- *Intersection*
- *Complementation*
- *Concatenation*
- *Star*

Closure Under Intersection

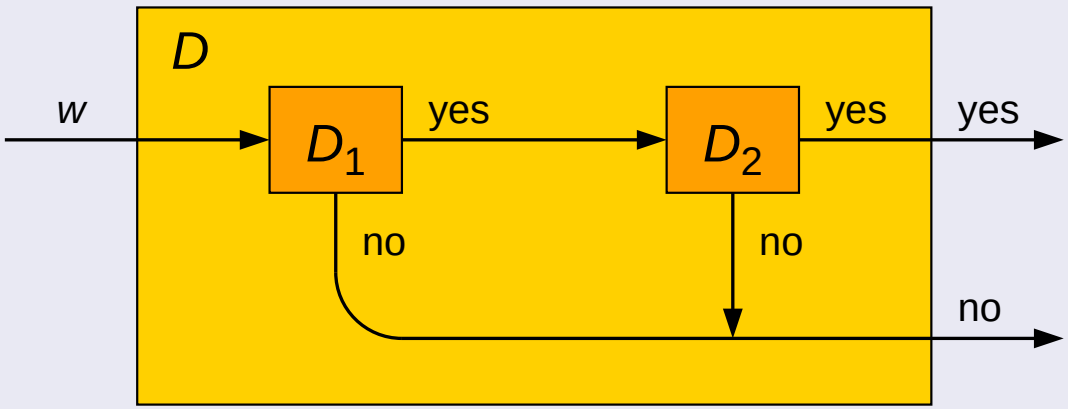
Theorem

If L_1 and L_2 are decidable, then $L_1 \cap L_2$ is decidable.

Closure of Intersection

Proof.

- Let D_1 be a decider for L_1 and let D_2 be a decider for L_2.
- Then build a decider D for $L_1 \cap L_2$ according to the following diagram.



Closure Under Union

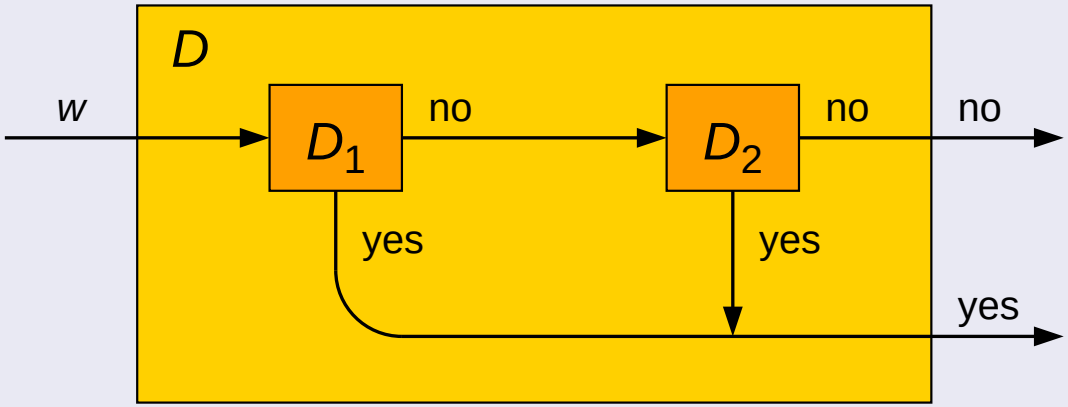
Theorem

If L_1 and L_2 are decidable, then $L_1 \cup L_2$ is decidable.

Closure Under Union

Proof.

- Let D_1 be a decider for L_1 and let D_2 be a decider for L_2.
- Then build a decider D for $L_1 \cup L_2$ according to the following diagram.



Closure Under Other Operators

- How would we show that if L_1 and L_2 are decidable, then so are
  - $\overline{L}_1$
  - $L_1 L_2$
  - L_1^*

Closure Properties of Recognizable Languages

Theorem (Closure Properties of Recognizable Languages)

The class of recognizable languages is closed under

- *Union*
- *Intersection*
- *Concatenation*
- *Star*

Closure Under Intersection

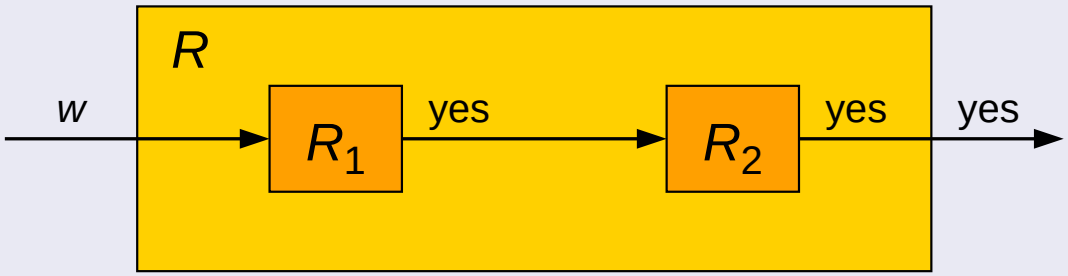
Theorem

If L_1 and L_2 are recognizable, then $L_1 \cap L_2$ is recognizable.

Closure Under Intersection

Proof.

- Let R_1 be a recognizer for L_1 and let R_2 be a recognizer for L_2.
- Then build a recognizer R for $L_1 \cap L_2$ according to the following diagram.



□

Closure Under Union

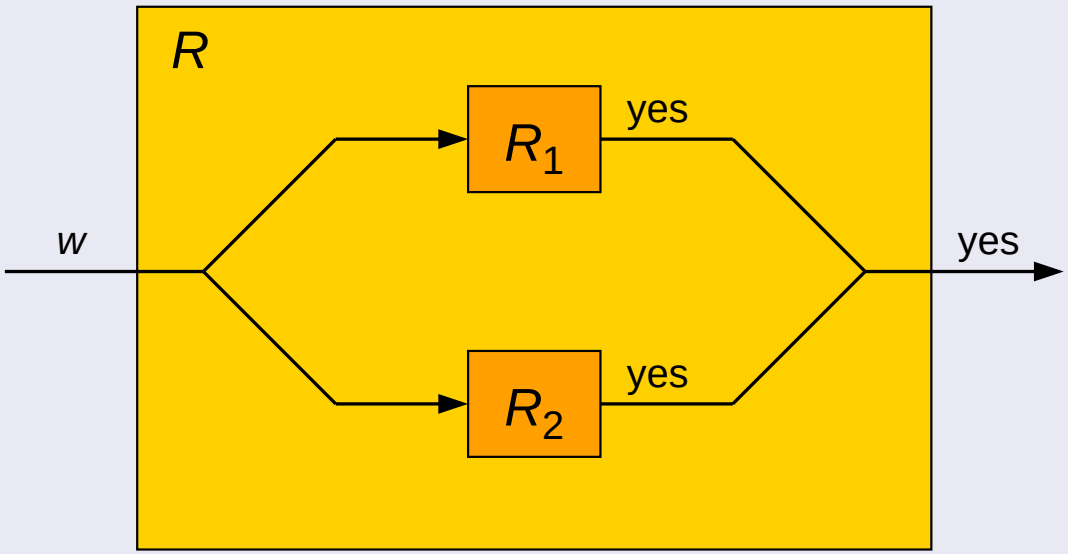
Theorem

If L_1 and L_2 are recognizable, then $L_1 \cup L_2$ is recognizable.

Closure Under Union

Proof.

- Let R_1 be a recognizer for L_1 and let R_2 be a recognizer for L_2.
- Then build a recognizer R for $L_1 \cup L_2$ according to the following diagram.



Closure of Union

Proof.

- In that diagram, we must be careful to alternate execution between R_1 and R_2.

□

Closure Under Other Operators

- How would we show that if L_1 and L_2 are recognizable, then so are
  - $L_1 L_2$
  - L_1^*
- Why is $\overline{L}_1$ not necessarily recognizable?

Assignment

Homework

- Read Section 3.2, pages 152 - 154.
- Problems 15, 16, page 161.