Finite Automata - Regular Operations

Lecture 7
Section 1.1

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Outline

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The Regular Operations

Definition (Union of languages)

The union of languages A and B is the language

$$A \cup B = \{w \mid w \in A \text{ or } w \in B\}.$$

Definition (Concatenation of languages)

The concatenation of languages A and B is the language

$$A \circ B = \{uv \mid u \in A \text{ and } v \in B\}.$$

Definition (Kleene star of a language)

The Kleene star of a language A is the language

$$A^* = \{w_1 w_2 \ldots w_k \mid w_i \in A \text{ and } k \geq 0\}.$$

The Regular Operations

- We often abbreviate $A \circ B$ as AB.
- Then we may abbreviate AA as A^2, AAA as A^3, and so on.
- The Kleene star of A can be written as

$$A^* = \{\varepsilon\} \cup A \cup A^2 \cup A^3 \cup \cdots .$$

Examples

Example (Regular operations)

- Let $A = \{w \mid w$ contains an even number of **a**'s$\}$.
- Let $B = \{w \mid w$ contains an even number of **b**'s$\}$.
- Describe the languages
  - $A \cup B$
  - $A \circ B$
  - A^*
  - $(A \cup B)^*$
  - $(A \circ B)^*$
  - $(A^*)^*$

Examples

Example (Regular operations)

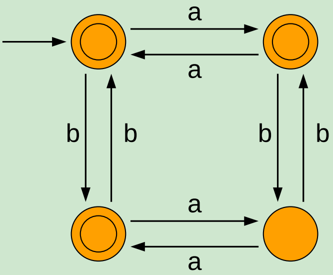
Design finite automata that accept

- $A \cup B$
- $A \circ B$
- A^*
- $(A \cup B)^*$
- $(A \circ B)^*$
- $(A^*)^*$

Examples

Example (Regular operations)

- A DFA for $A \cup B$.



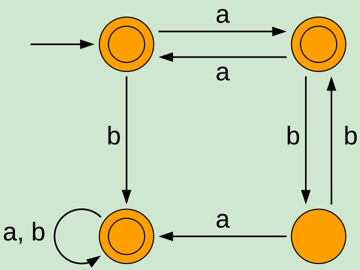
Examples

- Design a DFA for $A \cap B$.

Examples

Example (Regular operations)

- A DFA for $A \circ B$.



Outline

Closure

Theorem (Closure of Regular Languages)

The class of regular languages is closed under the operations of union, concatenation, and star.

Closure

Proof.

Proof (union)

- Let $M_1 = \{Q_1, \Sigma_1, \delta_1, q_1, F_1\}$ be a DFA whose language is L_1.
- Let $M_2 = \{Q_2, \Sigma_2, \delta_2, q_2, F_2\}$ be a DFA whose language is L_2.
- We will define a DFA M whose language is $L_1 \cup L_2$.
- Let $M = \{Q, \Sigma, \delta, q_0, F\}$ where
  - $Q = Q_1 \times Q_2$.
  - $\Sigma = \Sigma_1 \cup \Sigma_2$.
  - $q_0 = (q_1, q_2)$.
  - $F = \{(p_1, p_2) \mid p_1 \in F_1$ or $p_2 \in F_2\}$.

□

Closure

Proof.

Proof (union)

- Define $\delta : Q \times \Sigma \to Q$ by

$$\delta((p_1, p_2), a) = (\delta_1(p_1, a), \delta_2(p_2, a)).$$

- It is clear that the language of M is $L_1 \cup L_2$.

$\square$

Closure

Proof.

Proof (concatenation, star)

- What machine will accept $L_1 \circ L_2$?
- What machine will accept L_1^*?

□

Concatenation Example

Example (Concatenation Example)

- Let
$$L_1 = \{w \in \Sigma^* \mid w \text{ has an even number of } \mathbf{a}\text{'s}\}$$
and
$$L_2 = \{w \in \Sigma^* \mid w \text{ has an even number of } \mathbf{b}\text{'s}\}$$
- How would a DFA for $L_1 L_2$ process the strings **ababb** and **ababbb**?

Other Operations

Definition (Intersection)

The intersection of languages A and B is the language

$$A \cap B = \{w \mid w \in A \text{ and } w \in B\}.$$

Definition (Complement)

The complement of language A is the language

$$\overline{A} = \{w \in \Sigma^* \mid w \notin A\}.$$

Closure

Theorem (Closure of Regular Languages)

The class of regular languages is closed under the operations of intersection and complementation.

Closure

Proof.

Proof (intersection, complement)

- What machine will accept $L_1 \cap L_2$?
- What machine will accept $\overline{L_1}$?

□

Outline

Assignment

Homework

- Read Section 1.1, pages 44 - 47.
- Exercises 4, 5, 6, pages 83 - 84.
- Problem 34, page 89.
- Design a DFA for the language $(A \circ B)^*$, where

$$A = \{w \mid w \text{ contains an odd number of } \mathbf{a}\text{'s}\}$$
$$B = \{w \mid w \text{ contains an odd number of } \mathbf{b}\text{'s}\}$$