The Five-Number Summary

Lecture 16
Sections 5.3.1 - 5.3.3

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Outline

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2. Percentiles and Quartiles
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The salaries of superstar professional athletes receive much attention in the media.

The million-dollar annual contract is becoming more commonplace among this elite group with each passing year.

Nevertheless, rarely does a year pass without one or more of the players’ associations negotiating with team owners for additional salary and fringe-benefit considerations for all players in their particular sports.
Exercise 5.6, p. 311.

(a) If the players’ association wanted to support its argument for higher “average” salaries, which measure of center do you think it should use? Why?
(b) To refute the argument, which measure of center should the owners apply to the players’ salaries? Why?
Solution

(a) The players’ association should use the median. The distribution of salaries of professional athletes is skewed to the right (towards the larger values). Therefore, the median should be less than the mean.

(b) The owners should use the mean because it should be greater than the median.
Percentiles and Quartiles

Definition (\(p^{th}\) Percentile)

The \(p^{th}\) percentile of a set of numbers is a number that divides the lower \(p\%\) of the numbers from the rest.

Definition (1st Quartile)

The 1st quartile, denoted \(Q_1\), of a set of numbers is the \(25^{th}\) percentile.

Definition (3rd Quartile)

The 3rd quartile, denoted \(Q_3\), of a set of numbers is the \(75^{th}\) percentile.
Finding Quartiles

- To find the quartiles, first find the position of the median.
- Then the 1st quartile is the median of all the numbers that are below that position.
- The 3rd quartile is the median of all the numbers that are above that position.
Example

Example (Quartiles)

Find the median and quartiles of the following sample.

5, 8, 10, 15, 17, 19, 20, 24, 25, 30, 32
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Median
Example (Quartiles)

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Q₁, Median, Q₃
The five-number summary of a set of numbers consists of the five quantities:

- Minimum
- 1st quartile
- Median
- 3rd quartile
- Maximum

These five numbers divide the set of numbers into four groups of equal size, each containing one-fourth of the set.
Example (Five-Number Summary)

- The five-number summary of the previous sample is:
  - Min = 5.
  - \( Q_1 = 10. \)
  - Med = 19.
  - \( Q_3 = 25. \)
  - Max = 32.
Practice

Find the five-number summary of the sample

5, 8, 10, 15, 17, 19, 20, 24, 25, 30, 32, 35.
The Five-Number Summary

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Homework Review
Percentiles and Quartiles
Example
The Five-Number Summary
Examples
TI-83 Five-Number Summary

TI-83 Five-Number Summary

- Follow the same procedure that was used to find the mean.
- When the list of statistics appears, scroll down to the ones labeled minX, Q1, Med, Q3, maxX.
- They are the five-number summary.
### TI-83 Five-Number Summary

Use the TI-83 to find the five-number summary of the rainfall data

<p>| | | | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>2.82</td>
<td>24.18</td>
<td>0.20</td>
<td>15.60</td>
<td>22.04</td>
<td>7.44</td>
</tr>
<tr>
<td>5.16</td>
<td>9.14</td>
<td>37.36</td>
<td>10.19</td>
<td>2.16</td>
<td>17.50</td>
</tr>
<tr>
<td>28.12</td>
<td>11.23</td>
<td>8.66</td>
<td>7.24</td>
<td>6.50</td>
<td>4.88</td>
</tr>
<tr>
<td>13.08</td>
<td>4.01</td>
<td>11.28</td>
<td>1.96</td>
<td>12.09</td>
<td>2.92</td>
</tr>
<tr>
<td>7.67</td>
<td>4.39</td>
<td>6.60</td>
<td>6.50</td>
<td>25.43</td>
<td>0.74</td>
</tr>
</tbody>
</table>
If the distribution were uniform from 0 to 10, what would be the five-number summary?
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![Graph showing a uniform distribution from 0 to 10 with the median at 5.0.](image)
If the distribution were uniform from 0 to 10, what would be the five-number summary?

The diagram shows a number line from 0 to 10 with marks at 0, 1, 2, ..., 10 and percentiles at 25%, 25%, 25%, 25%. The quartiles are labeled: Q₁ (first quartile) at 3, Med (median) at 5, and Q₃ (third quartile) at 7.
Five-Number Summaries and Distributions

- Where would the median and quartiles be in this symmetric non-uniform distribution?
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Where would the median and quartiles be in this non-symmetric non-uniform distribution?
Where would the median and quartiles be in this non-symmetric non-uniform distribution?
Describe the distribution.

- Min
- $Q_1$
- Med
- $Q_3$
- Max
Five-Number Summaries and Distributions

- Describe the distribution.

\[ \text{Min} \quad Q_1 \quad \text{Med} \quad Q_3 \quad \text{Max} \]
Describing the distribution:

- Min
- $Q_1$
- Med
- $Q_3$
- Max
Describe the distribution.

- Min
- Q₁
- Med
- Q₃
- Max
The Interquartile Range

Definition (Interquartile Range)

The **interquartile range**, denoted IQR, is the difference between $Q_3$ and $Q_1$.

- The IQR is a commonly used measure of spread, or variability.
- Like the median, it is not affected by extreme outliers.
The IQR

Example (IQR)

The IQR of

\[ 5, 8, 10, 15, 17, 19, 20, 24, 25, 30, 32 \]

is

\[ \text{IQR} = Q_3 - Q_1 \]
\[ = 25 - 10 \]
\[ = 15 \]
The IQR

Practice

- Find the five-number summary and the IQR of the sample

\[5, 20, 30, 45, 60, 80, 100, 140, 175, 200, 240.\]

- Are the data skewed?
Five-Number Summaries and Stem-and-Leaf Displays

- Find a five-number summary of the following January rainfall data.

<table>
<thead>
<tr>
<th>Stem</th>
<th>Leaf</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0 0 1 2 2 2 4 4 4</td>
</tr>
<tr>
<td>0</td>
<td>5 6 6 6 7 7 8 9</td>
</tr>
<tr>
<td>1</td>
<td>0 1 1 2 3</td>
</tr>
<tr>
<td>1</td>
<td>5 7</td>
</tr>
<tr>
<td>2</td>
<td>2 4</td>
</tr>
<tr>
<td>2</td>
<td>5 8</td>
</tr>
<tr>
<td>3</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>7</td>
</tr>
</tbody>
</table>

- Note: 1|2 means 12.
## Salaries of School Board Chairmen

### Practice

Find the five-number summary of the following salaries of school board chairmen.

<table>
<thead>
<tr>
<th>County/City</th>
<th>Salary</th>
<th>County/City</th>
<th>Salary</th>
</tr>
</thead>
<tbody>
<tr>
<td>Henrico</td>
<td>20,000</td>
<td>Caroline</td>
<td>5,000</td>
</tr>
<tr>
<td>Chesterfield</td>
<td>18,711</td>
<td>Louisa</td>
<td>4,921</td>
</tr>
<tr>
<td>Richmond</td>
<td>11,000</td>
<td>Powhatan</td>
<td>4,800</td>
</tr>
<tr>
<td>Hanover</td>
<td>11,000</td>
<td>Hopewell</td>
<td>4,500</td>
</tr>
<tr>
<td>Petersburg</td>
<td>8,500</td>
<td>Charles City</td>
<td>4,500</td>
</tr>
<tr>
<td>Sussex</td>
<td>7,000</td>
<td>Prince George</td>
<td>3,750</td>
</tr>
<tr>
<td>New Kent</td>
<td>6,500</td>
<td>Cumberland</td>
<td>3,600</td>
</tr>
<tr>
<td>Goochland</td>
<td>5,500</td>
<td>King &amp; Queen</td>
<td>3,000</td>
</tr>
<tr>
<td>Dinwiddie</td>
<td>5,120</td>
<td>King William</td>
<td>2,400</td>
</tr>
<tr>
<td>Colonial Hgts</td>
<td>5,100</td>
<td>West Point</td>
<td>0</td>
</tr>
</tbody>
</table>
Excel’s Definition of Percentile

Definition (Excel’s $p^{th}$ percentile)

Excel’s $p^{th}$ percentile of a set of numbers is the number whose rank (position) is given by

$$r = 1 + \left( \frac{p}{100} \right) (n - 1).$$

If $r$ is not a whole number, then interpolate between values.

- Microsoft’s Excel uses a definition of the $p^{th}$ percentile that is based on the gaps between the numbers rather than on the numbers themselves.
Assignment

Homework

- Read Section 5.3.1 - 5.3.2, pages 312 - 315.
- Work Example 5.4, page 314, as an exercise.