Outline

1. Introduction
2. The $p$-Value Approach
3. The Hypothesis Testing Procedure
4. Example
   - The Hypotheses
   - The Significance Level
   - The Test Statistic
   - The Value of the Test Statistic
   - The $p$-Value
   - The Decision
   - The Conclusion
5. Hypothesis Testing on the TI-83
6. Assignment
Any question about a population must first be stated in terms of a population parameter.

We will work with only two parameters:
- The population mean $\mu$.
- The population proportion $p$. 
There are only two basic questions that we ask:

- What is the value of the parameter? (Estimation)
- Does the evidence support or refute a claim about the value of the parameter? (Hypothesis testing)
Example

If we want to learn about the effectiveness of a new drug...

- What parameter do we use?
- Do we estimate the parameter?
- Or do we test a hypothesis?
Example

If we want to find out whether a newborn child is more likely to be male than female...

- What parameter do we use?
- Do we estimate the parameter?
- Or do we test a hypothesis?
Outline

1 Introduction

2 The $p$-Value Approach

3 The Hypothesis Testing Procedure

4 Example
   - The Hypotheses
   - The Significance Level
   - The Test Statistic
   - The Value of the Test Statistic
   - The $p$-Value
   - The Decision
   - The Conclusion

5 Hypothesis Testing on the TI-83

6 Assignment
The $p$-Value Approach
The \( p \)-Value Approach
The *p*-Value Approach
The $p$-Value Approach

![Diagram showing the acceptance and rejection regions with a z-score and an alpha value.]

- **Acceptance Region**
- **Rejection Region**

- $p$-value
- $p$
- $\alpha$
- $c$

Robb T. Koether (Hampden-Sydney College)

Hypothesis Testing for Proportions

Tue, Mar 6, 2012 9 / 45
The *p*-Value Approach

\[ p \]

Acceptance Region

Rejection Region

Robb T. Koether (Hampden-Sydney College)
The $p$-Value Approach

- Acceptance Region
- Rejection Region
- $\alpha$

$0 \quad c \quad z$
The \( p \)-Value Approach

\[ p \]

Acceptance Region

Rejection Region

\[ z \]

\[ \alpha \]

\[ p \]

\[ 0 \]

\[ c \]

Acceptance Region

Rejection Region

Robb T. Koether (Hampden-Sydney College)  
Hypothesis Testing for Proportions  
Tue, Mar 6, 2012  
9 / 45
The \( p \)-Value Approach

\[ z \]

Acceptance Region

Rejection Region

\[ p_{\text{Accept}} \]

\[ 0 \quad z \quad c \]

Acceptance Region | Rejection Region

\[ \alpha \]
1. Introduction
2. The $p$-Value Approach
3. The Hypothesis Testing Procedure
   • The Hypotheses
   • The Significance Level
   • The Test Statistic
   • The Value of the Test Statistic
   • The $p$-Value
   • The Decision
   • The Conclusion
4. Example
5. Hypothesis Testing on the TI-83
6. Assignment
The seven steps of hypothesis testing.
The seven steps of hypothesis testing.

1. State the null and alternative hypotheses.
The Steps of Testing a Hypothesis

The seven steps of hypothesis testing.

1. State the null and alternative hypotheses.
2. State the significance level.
The seven steps of hypothesis testing.

1. State the null and alternative hypotheses.
2. State the significance level.
3. State the formula for the test statistic.
The seven steps of hypothesis testing.

1. State the null and alternative hypotheses.
2. State the significance level.
3. State the formula for the test statistic.
4. Compute the value of the test statistic.
The seven steps of hypothesis testing.

1. State the null and alternative hypotheses.
2. State the significance level.
3. State the formula for the test statistic.
4. Compute the value of the test statistic.
5. Compute the $p$-value.
The seven steps of hypothesis testing.

1. State the null and alternative hypotheses.
2. State the significance level.
3. State the formula for the test statistic.
4. Compute the value of the test statistic.
5. Compute the $p$-value.
6. Make a decision.
The seven steps of hypothesis testing.

1. State the null and alternative hypotheses.
2. State the significance level.
3. State the formula for the test statistic.
4. Compute the value of the test statistic.
5. Compute the $p$-value.
6. Make a decision.
7. State the conclusion.
Outline

1 Introduction

2 The $p$-Value Approach

3 The Hypothesis Testing Procedure

4 Example
   - The Hypotheses
   - The Significance Level
   - The Test Statistic
   - The Value of the Test Statistic
   - The $p$-Value
   - The Decision
   - The Conclusion

5 Hypothesis Testing on the TI-83

6 Assignment
Example (Hypothesis testing)

- We want to test a coin for fairness?
- That is, does it land heads half the time?
- Suppose a random sample of 1000 coin tosses produces 525 heads and 475 tails.
- Test the hypotheses that coin is fair vs. not fair, at the 5% level of significance.
Outline

1. Introduction
2. The \( p \)-Value Approach
3. The Hypothesis Testing Procedure
4. Example
   - The Hypotheses
   - The Significance Level
   - The Test Statistic
   - The Value of the Test Statistic
   - The \( p \)-Value
   - The Decision
   - The Conclusion
5. Hypothesis Testing on the TI-83
6. Assignment
The Hypotheses

- Select the appropriate parameter \((p)\) and describe what it represents.
- The null hypothesis should state a hypothetical value \(p_0\) for the population proportion.

\[ H_0 : p = p_0. \]
The alternative hypothesis must contradict the null hypothesis in one of three ways:

- $H_1 : p < p_0$. (if direction of extreme is left.)
- $H_1 : p > p_0$. (if direction of extreme is right.)
- $H_1 : p \neq p_0$. (if direction of extreme is left and right.)
Example (Step 1)

(1) Let $p =$ proportion of tosses that land heads.

$H_0 : p = 0.50.$

$H_1 : p \neq 0.50.$
Outline

1 Introduction
2 The p-Value Approach
3 The Hypothesis Testing Procedure
4 Example
   • The Hypotheses
   • The Significance Level
   • The Test Statistic
   • The Value of the Test Statistic
   • The p-Value
   • The Decision
   • The Conclusion
5 Hypothesis Testing on the TI-83
6 Assignment
Specify the level of significance $\alpha$. 
Example (Step 2)

(2) $\alpha = 0.05$. 
1 Introduction

2 The $p$-Value Approach

3 The Hypothesis Testing Procedure

4 Example
   • The Hypotheses
   • The Significance Level
   • The Test Statistic
     • The Value of the Test Statistic
     • The $p$-Value
   • The Decision
   • The Conclusion

5 Hypothesis Testing on the TI-83

6 Assignment
A **test statistic** is the statistic that is used to make the decision in a hypothesis test.
The Test Statistic

Definition (Test statistic)

A test statistic is the statistic that is used to make the decision in a hypothesis test.

- What should the test statistic be in this situation?
According to the Central Limit Theorem, the statistic \( \hat{p} \) has a normal distribution with

\[
\mu_{\hat{p}} = p
\]

and

\[
\sigma_{\hat{p}} = \sqrt{\frac{p(1-p)}{n}}.
\]
Therefore, if the null hypothesis is true, then \( \hat{p} \) is normal with mean \( p_0 \) and standard deviation \( \sqrt{\frac{p_0(1-p_0)}{n}} \).
Therefore, if the null hypothesis is true, then \( \hat{p} \) is normal with mean \( p_0 \) and standard deviation \( \sqrt{\frac{p_0(1-p_0)}{n}} \).

We will use the \( z \)-score of \( \hat{p} \), which is computed as

\[
z = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}}.
\]
Therefore, if the null hypothesis is true, then $\hat{p}$ is normal with mean $p_0$ and standard deviation $\sqrt{\frac{p_0(1-p_0)}{n}}$.

We will use the $z$-score of $\hat{p}$, which is computed as

$$z = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}}.$$ 

The $z$-score has a standard normal distribution.
Example (Step 3)

(3) The test statistic is

\[ z = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}} \]
1. Introduction
2. The \( p \)-Value Approach
3. The Hypothesis Testing Procedure
4. Example
   - The Hypotheses
   - The Significance Level
   - The Test Statistic
   - The Value of the Test Statistic
   - The \( p \)-Value
   - The Decision
   - The Conclusion
5. Hypothesis Testing on the TI-83
6. Assignment
To compute the value of the test statistic, substitute the values obtained from the sample and from the null hypothesis.

In this case, use the values of $\hat{p}$, $p_0$, and $n$. 
Example (Step 4)

From the null hypothesis, we have $p_0 = 0.50$.

$$p_0 = 0.50,\quad \hat{p} = \frac{525}{1000} = 0.525,\quad n = 1000,$$

Compute

$$Z = \frac{0.525 - 0.50}{\sqrt{(0.50)(1-0.50) \cdot \frac{1}{1000}}} = \frac{0.025}{0.01581} = 1.581.$$
Introduction

The $p$-Value Approach

The Hypothesis Testing Procedure

Example

- The Hypotheses
- The Significance Level
- The Test Statistic
- The Value of the Test Statistic
- The $p$-Value
- The Decision
- The Conclusion

Hypothesis Testing on the TI-83

Assignment
The $p$-Value

- To find the $p$-value, use the `normalcdf` function and the value of the test statistic ($z$).

- Pay attention to the direction of extreme as indicated by the alternative hypothesis.
  - To the left: $p$-value $= \text{normalcdf}(-E99, z)$.
  - To the right: $p$-value $= \text{normalcdf}(z, E99)$.
  - Two-sided: Find the area in the appropriate tail and then double it.
Example (Step 5)

(5)

\[ p\text{-value} = 2 \times \text{normalcdf}(1.581, E99) = 2 \times 0.0569 = 0.1138. \]
Outline

1. Introduction
2. The $p$-Value Approach
3. The Hypothesis Testing Procedure
4. Example
   - The Hypotheses
   - The Significance Level
   - The Test Statistic
   - The Value of the Test Statistic
   - The $p$-Value
   - The Decision
   - The Conclusion
5. Hypothesis Testing on the TI-83
6. Assignment
The decision states whether to accept or reject the null hypothesis.

If the $p$-value is less than $\alpha$, then write “Reject $H_0$.”

If the $p$-value is greater than $\alpha$, then write “Accept $H_0$.”
Example (Step 6)

(6) Accept $H_0$. 

Outline

1. Introduction
2. The $p$-Value Approach
3. The Hypothesis Testing Procedure
4. Example
   - The Hypotheses
   - The Significance Level
   - The Test Statistic
   - The Value of the Test Statistic
   - The $p$-Value
   - The Decision
   - The Conclusion
5. Hypothesis Testing on the TI-83
6. Assignment
The conclusion restates the decision *in the language of the original problem*, without using any statistical jargon.

It is enough to restate in plain English the hypothesis that was accepted.
Example (Step 7)

(7) The proportion of heads is 50%.
(8) Or, the coin is fair.
Example (The seven steps)
(1) Let $p =$ proportion of tosses that land heads.
   $H_0 : p = 0.50$.
   $H_1 : p \neq 0.50$.
(2) $\alpha = 0.05$.
(3) The test statistic is $Z = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}}$.
(4) $Z = \frac{0.525-0.50}{\sqrt{\frac{(0.50)(1-0.50)}{1000}}} = \frac{0.025}{0.01581} = 1.581$.
(5) $p$-value $= 2 \times \text{normalcdf}(1.158, E99) = 0.1138$.
(6) Accept $H_0$.
(7) The proportion of heads is 50%.
Outline

1. Introduction
2. The $p$-Value Approach
3. The Hypothesis Testing Procedure
4. Example
   - The Hypotheses
   - The Significance Level
   - The Test Statistic
   - The Value of the Test Statistic
   - The $p$-Value
   - The Decision
   - The Conclusion
5. Hypothesis Testing on the TI-83
6. Assignment
TI-83 Hypothesis Testing for $p$

- Press **STAT**.
- Select the **TESTS** menu.
- Select **1-PropZTest**...
- Press **ENTER**. A window appears.
- Enter the value of $p_0$.
- Press **ENTER** and the down arrow.
- Enter the numerator $x$ of $\hat{p}$.
- Press **ENTER** and the down arrow.
- Enter the sample size $n$.
- Press **ENTER** and the down arrow.
Hypothesis Testing on the TI-83

TI-83 Hypothesis Testing for $p$

- Select the type of alternative hypothesis.
- Press the down arrow.
- Select Calculate.
- Press ENTER.
TI-83 Hypothesis Testing for \( p \)

- The following appear in the display.
  - The title \( 1\text{-PropZTest} \).
  - The alternative hypothesis.
  - The value of the test statistic \( Z \).
  - The \( p \)-value.
  - The value of \( \hat{p} \).
  - The sample size \( n \).
TI-83 Example

TI-83 $p$-value approach

- Work the example of 525 heads and 475 tails on the TI-83.
Outline

1 Introduction
2 The $p$-Value Approach
3 The Hypothesis Testing Procedure
4 Example
   • The Hypotheses
   • The Significance Level
   • The Test Statistic
   • The Value of the Test Statistic
   • The $p$-Value
   • The Decision
   • The Conclusion
5 Hypothesis Testing on the TI-83
6 Assignment
Assignment

Homework

- Exercises 1 - 12, 14, page 580.