Algebraic Money: Berkeley’s Philosophy of Mathematics and Money

C. George Caffentzis

Our spinning school is in a thriving way. The children begin to find a pleasure in being paid in hard money.¹

In the early 1730s George Berkeley began to explore the conceptual field between ideas and spirits that he previously claimed to be empty. In this field he found a rich set of concepts including “notions,” “principles,” “beliefs,” “opinions,” and even “prejudices.” Elsewhere I have referred to this phase in Berkeley’s thought as his “second conceptual revolution.”² I believe that it was motivated by his increasing need to develop a language to discuss the social, moral and theological concerns vital to him and his circle.

This second conceptual revolution made possible two of his most important contributions to 18th century thought: The Analyst (1734) and The Querist (1735-37). Even though they were written almost simultaneously, these texts are rarely discussed together, since the former is categorized as a critique of the foundations of the calculus, while the latter is taken a tract advocating the development of a specie-less economy in Ireland. Using new textual and contextual evidence, however, I will show with that these two texts have a common basis in Berkeley’s second conceptual revolution, in that the rejection of intrinsic values (either epistemic or monetary) and the revaluation of notions, principles, and prejudices are crucial to the critique of both Newtonian mathematics in The Analyst and Newtonian monetary theory and policy in The Querist.

Specifically, I will argue that Berkeley’s famous demonstration of the absurdities of Newton’s method of fluxions devalued geometric reasoning and gave a new pride of place to algebraic reasoning. On the basis of this revaluation in mathematics, Berkeley more confidently undermined the concept of intrinsic monetary value and suggested the development of a monetary system based on “tickets, tokens and counters” (what I call “algebraic money”).

The issues posed by the transition from a specie-based to a specie-less currency were clearly some of the most important and controversial in the Age of Enlightenment. Berkeley’s contributions to understanding the significance and feasibility of such a

transition and its benefits for Ireland certainly add support the claim that he was “the most engaging and useful man in Ireland in the eighteenth century.”

I. The Analyst and The Querist as Products of Berkeley’s “Second Conceptual Revolution”

In 1734 and 1735 George Berkeley began a new phase in his ecclesiastic career by becoming the Bishop of Cloyne. In those years, he also wrote and published (with the help of friends like Samuel Madden, Thomas Prior, and Lord Percival) two important pamphlets, The Analyst and The Querist. One dealt with mathematics and the other with money, but both had profound consequences on their respective fields and are often cited in histories of mathematics and economics. They are, however, rarely examined comparatively. This lacuna in the literature on Berkeley’s writings is strange for two reasons.

First, these two works are products of an important moment in Berkeley’s conceptual creativity (his “second conceptual revolution”) and in the development of his political and social sensibilities. One would be surprised, for example, that such a politically aware figure like Berkeley, who for almost a decade lobbied Parliament and the Queen in a failed effort to fund his utopian multi-racial college in the Bermudas, did not recognize that The Analyst and The Querist put into question the work of the Whig establishment’s intellectual centerpiece, Isaac Newton—both as a mathematician (in his role as “the Great Author” of the method of fluxions) and as the Master of the Mint (in his role as an originator of the gold standard). But what for Berkeley might have appeared as a seamless critical connection between these two works has not been noticed in a commentary literature that still seems to be ruled by disciplinary rubrics.

Second, there is a long and insightful interpretive tradition that has connected changes in mathematical thought with transformations in monetary reality (and vice versa). Participants in this tradition include the founders of modern social thought like Karl Marx and Georg Simmel as well as their more recent heirs such as Alfred Sohn-Rethel and Joel Kaye.³ The claim that money stimulated the mathematization of the social (and natural) world, and that mathematics makes possible the monetarization of the social (and natural) world is, of course, an essential theme in the philosophy of money. Adherents of this interpretive tradition would immediately suspect that there would be significant intertextual transformations between The Analyst and The Querist worth noting. And that is what I highlight here.

After some general comments on these themes, I will provide an important example of how The Analyst and The Querist are related. I argue that Berkeley’s critique of the

application of geometric representation to the defense of “analysis” and his allied revaluation of algebraic representation in The Analyst are implicated in and support the conception of money developed in The Querist.

Of course, the fact that Berkeley wrote The Analyst and The Querist at around the same time does not automatically justify treating them comparatively. But other evidence, both contextual and textual, supports the hypothesis that both of these works constitute elements of a common project. In this section I examine two pieces of this evidence: (a) the importance of “notions” in the development of Berkeley’s philosophy in the 1730s, including in both The Analyst and The Querist; and (b) the identity of the antagonists addressed in both texts: infidel mathematicians (e.g., the Royal Astronomer and Savilian Professor of Geometry, Edmond Halley), followers of Newton, libertines, and atheists.

Regarding Berkeley’s introduction of his doctrine of notions, it is useful to recognize how he was something of a philosophical prodigy, and he paid for it. He published major (and minor) works on mathematics, vision, and philosophy between his 22nd (1707) and 28th (1713) years that clearly identified him politically as a Tory and intellectually as a clever but harsh critic of materialism, libertinism, and atheism. His critique of these tendencies was based on the dichotomy he drew between ideas and spirits. Ideas were passive and detached, while spirits were active, creative, and capable, among other things, of making relations among ideas. Most important, one could neither have ideas about spirits nor describe spirits and their operations (e.g., willing, loving, hating) using words that refer to ideas. This limitation, however, was needed in order to get the quick results he desired—namely, “proof” of the inconceivability of unperceived things or objects and “proof” of the contradictory character of matter.4

However, just as Berkeley had to pay for his early identification with the Harley–St. John Tory regime (1710-1714) by spending the rest of his life under the cloud of “Jacobitism,” he also later had to deal with puzzling aspects of his initially successful philosophic program. Given the constraints of the idea/spirit dichotomy, he could not carry on a legitimate discourse concerning the life of spirits unless he found an appropriate referent for the part of language that is normally taken to refer to spirits and their operations, including their relation-making ability and products. To do this he introduced a specific term of art, “notion,” to distinguish it from “idea.” Previously he often used “notion” as synonymous with “idea,” but with the revision of the Principles and Dialogues in 1734, he granted that though he had no idea of the words will, soul, and spirit: “we have some

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4 An early “proof” of the falsehood of “the opinion strangely prevailing amongst men, that houses, mountains, rivers and in a word all sensible objects have an existence natural or real, distinct from their being perceived by the understanding” is achieved in Berkeley’s Treatise on the Principles of Human Knowledge, sec. 4. See George Berkeley, Philosophical Works Including the Works on Vision, ed. Michael R. Ayers (London: Everyman, 1993), 90. [Hereafter: PHK and section number; or in the case of the Principles Introduction, PHK IN section.]
notion of soul, spirit, and the operations of the mind, such as willing, loving, hating, in as much as we know or understand the meaning of these words” (PHK 27).

This admission of a capacity of words to signify non-ideas in a meaningful way had many positive (and a few negative) consequences for Berkeley. Its primary benefit was that it made it possible to carry on sophisticated discourse about the “soul, spirit, and the operations of the mind” consistently—not a trivial detail for a bishop whose job was to minister to such souls! But it also confronted him with a challenge of policing the range of notional entities that were in their nature unimaginable.

When he was dealing with notions he could not with confidence use his old critical experimental technique of claiming that, since he had no idea of x and doubted whether anyone else did, x-talk was illegitimate. This “subjective empiricist” technique worked splendidly for a critique of abstract ideas. But such a test could not apply to notions, since they cannot be experienced or perceived in the same way as ideas. Accordingly, the act of willing and its actor (which one has notions about) are literally un-imagine-able. Without such an introspective test (i.e., either I have that idea or I don’t), how can one set the bar on the existence of notions, including the fluxion?

Toward the end of The Analyst Berkeley reveals that, “Of a long time I have suspected that these modern analytics were not scientifical, and gave some hints thereof to the public twenty-five years ago.” But his older 1710 test for the existence of ideas—“if therefore I cannot perceive innumerable parts in any finite extension that I consider, it is certain they are not contained in it”—does not work for notions in 1734, since being perceived is not a criterion of their existence (PHK 124). Indeed, to have perceived a notion (in the technical sense) is in itself a contradiction!

During Berkeley’s “second conceptual revolution,” notions become more central to his thought, and The Analyst and The Querist share his concerns about them. In The Analyst Berkeley is concerned about the use of notional terminology to mask logical and

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5 Another lengthy 1734 addition to the Principles that expands on this point is: “We may not I think strictly be said to have an idea of an active being, or of an action, although we may be said to have a notion of them. I have some knowledge or notion of my mind and its acts about ideas, inasmuch as I know or understand what is meant by those words. What I know, that I have some notion of. . . . It is also to be remarked, that all relations including an act of the mind, we cannot so properly said to have an idea, but rather a notion of the relations and habitudes between things” (PHK 142). But I should point out that Berkeley never completely standardized the technical distinction between ideas and notions into a strict linguistic dichotomy in his work. He often used terms like “idea or notion” (for example, “of Extension prior to Motion” in Querist 12, and he used the phrase “true Idea of money” in The Querist to refer to what I have called his “notional” conception of money. [Citations from The Querist (by section) are taken from Bishop Berkeley’s “Querist” in Historical Perspective, ed. Joseph Johnston (Dundalk, Ireland: Dundalgan Press, 1970).] For an excellent discussion of Berkeley’s doctrine of notions, see Daniel E. Flage, Berkeley’s Doctrine of Notions: A Reconstruction based on His Theory of Meaning (New York: St. Martin’s Press, 1987).

6 See David Berman, Berkeley: Experimental Philosophy (New York: Routledge, 1999), 5-10.

conceptual absurdities. He uses the term “notion” often in the text and he explicitly categorizes the suspect fluxion as a notion (cf. Analyst 10, 38–40). Consequently, one can see The Analyst as an attempt to police the notion of “notion” in mathematics.

In contrast, in The Querist “notion” is utilized as the intellectual basis of money, in that the crucial aspect of money is not that it truly signifies some idea (e.g., an ounce of gold), but that it excites the players in the monetary game to industry, that is, to work and to invest productively. As Berkeley rhetorically queries:

> Whether it be not the opinion or will of the people, exciting them to industry, that truly enricheth a nation? And whether this doth not principally depend on the means for counting, transferring, and preserving power, that is, property of all kinds? (Querist 31)

As a consequence, the key issues concerning money involve will, action and power; all of these are fundamentally notional entities. Thus the truth of monetary signs is not, as Locke would have it, whether coins accurately contain the metal they claim on their face. As Berkeley sees it, in the realm of money the distinction between “real” and the “notional” is not crucial:

> Whether the opinion of men, and their industry consequent thereupon, be not the true wealth of Holland and not the silver supposed to be deposited in the Bank at Amsterdam? (Querist 44)

> Whether there is in truth any such treasure lying dead? And whether it be of great consequence to the public that it should be real rather than notional? (Querist 45)

But the concerns generated by his notional conception of money made Berkeley clarify to his readers the need for precautions against abuse that would be largely irrelevant when dealing with a specie-dominated money. As I have expressed it elsewhere, readers might have argued with some justice that “the Querist’s enlightened liberation from the superstitious magic of Gold and Silver opened up so many possibilities for arbitrary, willful manipulation of the currency that it was best to stay with the old, chaotic, but relatively abuse-proof system.” Berkeley had to assure his readers that his National Bank would be designed to effectively police the notional character of money and a large part of The Querist is devoted exactly to describing mechanisms for the task.

In this way, both The Analyst and The Querist evince their participation in the increasing importance of notional entities in Berkeley’s work at the time. But there are other commonalities worth noting, most obviously of which is an overlap in the open and hidden antagonists of both texts. Antagonists were very important for Berkeley. In this regard, his temperament can be deceptive. By all accounts he was a sweet, calm and welcoming person. As an intellect, however, he was polemical. His work is often

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9 Caffentzis, Exciting the Industry, 299.
conceived and produced in the context of bitter ideological struggle. This disposition cost him much, including the most prized project of all, the multi-racial St. Paul’s College in the Bermudas. During most of his productive life, his “extremist” texts inspired antagonism and suspicion in a Whig-dominated government. Moreover, in this struggle, he “took no prisoners.” In fact, he was positively draconian when it came to imagining punishments for his enemies, the libertines, atheists, and blasphemers.10

Consequently, in Berkeley studies it is important to “know the enemy,” although it is often not easy to “name names.” Berkeley believed that he and his Church lived through perilous times and faced many mortal enemies (whom he was often loath to name). Certainly these enemies would include the late 17th century Whig Junto in London that lived on and triumphed in the more than twenty years of the Robinocracy. There was also a powerful opposition in Ireland among the gentry whose representatives in the Irish Commons had voted to stop paying the agistment tithe (a tithe on cattle) to the Anglican Church of Ireland in 1734. Berkeley believed that libertines were agitating against paying tithes and gave ideological support to the tithe revolt.11

However, the intellectual figure who stands out as Berkeley’s antagonist in both *The Analyst* and *The Querist* is Newton. Newton was an important part of the Whig intelligentsia, and he led the chief public organs of mathematics and money, the Royal Society and the Royal Mint, for the first quarter of the 18th century. Moreover, he was the “inventor” of the method of fluxions and the “inaugurator” of the gold standard.12 Of course, in *The Analyst* Newton is referred to not as the living “mathematical infidel” addressed in the text (after all Newton had been dead for seven years by the time of publication), but as “the Great Author” of the notion of the fluxion.13 In *The Querist*
Berkeley does not refer directly to the former Master of the Mint; but anyone who wrote of specie in Britain during this period had to deal with the work and legacy of Newton.\textsuperscript{14}

Berkeley had a very complex relation with Newton. He did recognize that Newton was not an atheist (although there were many rumors circulating about his dismissal of Trinitarianism and his possible monophysitism).\textsuperscript{15} He also recognized the importance of Newton’s *Principia*, for all his philosophical differences with Newton’s notions of absolute space and time. Indeed, Newton’s interventionist Pancrator conception of God has some resemblance to the Berkeley’s loquacious God who continually delivered sensory ideas for his creatures.

But emanating out from Newton were ever-enlarging and overlapping fields of opponents including “The Great Author’s” followers (e.g., Halley), whom he called “Philomathematical Infidels of these Times,” libertines, free-thinkers, and materialists, even though Newton—being somewhat like the absent-minded but pernicious king of Laputa in Swift’s *Gulliver’s Travels*—perhaps was not a member of any of these sects. This common source of evil, according to Berkeley’s lights, would inevitably bring one to see a common problematic posed by *The Analyst* and *The Querist*.

\textsuperscript{14} Strangely, this receives no attention in *The Cambridge Companion to Newton*.

\textsuperscript{15} Newton’s anti-Trinitarianism was something of an open secret among his friends (and a few of his enemies). Newton struggled to keep it *sub rosa*. As Scott Mandelbrote writes: “For Newton, the notion of the divine Trinity represents the culmination of the human tendency to corrupt religion into idolatry. . . . Newton wanted to confine suffering for his faith to the private experience of his closet, even though his personal beliefs were quite different from those of ordinary members of the Church to which he nominally belonged” (Mandelbrote, “Newton and Eighteenth-century Christianity,” in *Cambridge Companion to Newton*, 421).
II. Money and Mathematics: A Précis of the History of the Relation and Its Application to Our Case

In the previous section I claimed that there are biographical reasons to compare *The Querist* and *The Analyst*—namely, the insistence on notional elements as the means for overcoming problems posed in theories of money and mathematics, and the common antagonists in each of these two areas. In this section I will deal with the wider set of connections between money and mathematics that lead me to suspect that there are many shared themes in these two texts.

Much 19th and early 20th century social theory centers on how the development of the form of money has had a profound influence of mathematics and vice versa. The most insightful commentator on this interaction was Georg Simmel, who argued that with the triumph of a monetary society, social life inevitably became both more mathematical and “intellectual,” in the sense that the participants in a monetarized economy are continually posing and confronting complex and ever lengthening series of means to achieve their ends.

Simmel’s *Philosophy of Money* (1900) is, in fact, a paean (and a dirge) that addresses the impact of money on social life. In his characteristic suggestive prose he writes in the section “The calculating character of modern times”:

> By and large, one may characterize the intellectual functions that are used at present in coping with the world and regulating both individual and social relations as calculative functions. Their cognitive ideal is to conceive of the world as a huge arithmetical problem, to conceive events and the qualitative distinction of things as a system of numbers. . . . The money economy enforces the necessity of continuous mathematical operations in our daily transactions. The lives of many people are absorbed by such evaluating, weighing calculating and reducing of qualitative values to quantitative ones. (444)

But the mathematization of everyday life as a consequence of its monetarization is only the more obvious, often decried, aspect of the relationship between mathematics and money. According to Simmel, there is a more subtle, transcendental connection to be made: money creates the foundations for a mathematical conceptualization of value tout court, because it makes stable, reified, and objective values possible. The possibility of applying mathematics (be it arithmetic, geometry, or algebra) to human affairs necessitates a set of values that have these characteristics; otherwise there would be no point in attempting to apply mathematics to them or to reflect mathematically on them. If one lived in an Alice-in-Wonderland dream world, where a set of six “entities” and a distinct set of five “entities” are merged inexplicably into one of twelve “entities,” then both the scare quotes and the rules of addition would be useless in practice. In Simmel’s account, money, which is the ultimate product of economic exchange, in effect creates “a realm of values that is more or less completely detached from the subjective-personal substructure,” even though it arises from it (79). A mathematics (as well as a logic and
law) of human activity can only develop in a world of absolute values that are created in a monetary society.

Simmel’s insight (as well as Marx’s) had an impact on the history of mathematics in the twentieth century (as well as in the philosophy of mathematics via the Hegelian-Marxist influenced work of Imre Lakatos and social constructivism). Historical research on the major fluorescent periods of mathematics (and mathematical physics) was revised in its light. Thus, the connection between the beginning of coinage in Lydia in the 7th century B.C.E. and the development of geometry and other forms of mathematics in Magna Grecia in the following two centuries has become something of a well-traveled road. More recently, Joel Kaye has studied the tie between the revival of economic and monetary life and the development of a new mathematical physics in the later medieval period. He aims to “provide an outline of a mechanism of transference between the scholar’s conception of the social world and his conception of the natural world, between his insights into the working of a monetized society and his insights into the working of a newly quantifiable and measurable nature.”

Of course, the interrelation of the rise of capitalism in the 15th and 17th centuries with the “mathematization of the world” of that period is the most developed site in this tradition of scholarship. The theory of probability has received the bulk of recent attention from this perspective, but the development of the calculus has often been connected with the impact of monetarization as well.

This research program leads me to suspect that Berkeley’s almost simultaneous composition of The Analyst and The Querist was no accident, and that there are important cross-references and common themes between the economic and mathematical aspects of these works (as well as the “economic” and mathematical and philosophical work of Berkeley’s immediate 17th antecedents and 18th century contemporaries). Accordingly, I

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18 Kaye, Economy and Nature, 12.


will follow its path and in the following section I will present a common theme I have
discovered: Berkeley’s revaluation of algebraic representation and its use in monetary
theory and practice.

III. Algebra :: Paper
     Geometry :: Specie

I have given some reasons why I believe that there are important common themes in both
*The Querist* and *The Analyst* above, but one can with justice say that there are other
reasons to keep them separate. After all, *The Analyst* deals with recondite technical
details in analysis (what is now largely called “the differential and integral calculus”) that
were at the frontiers of mathematical practice in the first part of the 18th century, while
*The Querist* is a critique of metallism (both theoretical and practical). *The Analyst* does
not deal with money, while *The Querist* does not deal with the calculus; so why should
they meet?

I argue that there is a central theme that both texts share—*representation*—and that both
*The Analyst* and *The Querist* announce (a) a crisis of representation in their respective
fields, (b) a critique of self-reflexive forms of representation, (c) a need to overcome the
crisis by a revaluation of current systems of representation, and (d) the importance of
algebraic methods in both mathematical and monetary representation.

*The Analyst* and *The Querist* deal with contrasting pairs of systems of representation:
algebraic vs. geometric representation in the former, and specie vs. paper forms of money
in the latter. Although in practice algebra and geometry since Descartes’ *Geométrie* were
being used cooperatively, algebraic and geometric “ideologies” were often in conflict
with each other among mathematicians as well as philosophers. 21 For example, as
Douglas Jesseph points out, “Hobbes is famous for his rejection of the methods of
symbolic algebra as a ‘scab of symbols’ which deface geometric demonstrations” (120);
and Hobbes was not alone. 22 Indeed, this algebra/geometry tension was very much in
evidence in Berkeley’s *Analyst* and has deep roots in his own thought. 23

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21 See Douglas M. Jesseph, *Berkeley’s Philosophy of Mathematics* (Chicago: University of

22 Newton in his last days was as concerned about this “scab” as was Hobbes almost a century
before. Henry Pemburton, the editor of the third [1726] edition of the *Principia*, wrote: “I have
often heard him [Newton] censure the handing of geometrical subjects by algebraic calculations. .
. Of their [the ancients’] taste and form of demonstration Sir Isaac always professed himself a
great admirer: I have heard him even censure himself for not following them yet more closely
than he did, and speak with regret of his mistake at the beginning of his mathematical studies in
applying himself to the works of Des Cartes and other algebraic writers before he had considered
the elements of Euclide with that attention, which so excellent a writer deserves” (cited in
Companion to Newton*, 318).

Similarly, in practice, even though most economies in the 18th century had both precious metal coinage and various forms of paper currency operating (and interchanging) side by side, the metallic and paper “ideologies” were often in conflict with each other. Indeed, the whole point of Berkeley’s Querist is to stimulate interest in creating a specie-less currency in Ireland due to his conviction that specie operated as a harmful “drug” there.

III.a The Dual Crises

The Analyst and The Querist were implicated in the dual “wars” of the representational systems of both mathematics and money which, as far as Berkeley was concerned, had reached crisis proportions in the early 1730s. The crisis of representation to which The Analyst points is of a system of representation that had enormous prestige in mathematics (viz., geometry), but for Berkeley that system was being subverted by mathematicians who were “infidels” not only against religion but also against the ideals of geometry itself. As Berkeley had emphasized in his early mathematical writings, geometric representation is rooted in perceivable extension, and its signifiers are diagrams that have immediate similitude with their signified. Geometric demonstrators employ diagrams to keep their selective attention grounded in making their proofs about the lines and figures of the science. As a consequence, the early Berkeley admired the power of geometric reasoning especially as a pedagogical tool.

However, with the development of the calculus and Newton’s allied “method of fluxions” as its justification, Berkeley began to question the authority of geometric representation. He writes in The Analyst: “Of a long Time I have suspected, that these modern Analytics were not scientifical, and gave some Hints thereof to the Public about twenty five years ago” (Analyst 50). For he believed, according to Jesseph:

the calculus is fundamentally a geometric theory, whose proper object is perceivable extension. Thus, the key terms in the calculus must be interpretable in terms of perceivable extension, i.e., we must be able to frame ideas corresponding to these terms. . . . a theorem of the calculus (such as the determination of the arc-length of a curve) concerns extended objects and cannot be legitimately obtained unless each step in the derivation has the appropriate ideas corresponding to it. (Jesseph 116-17)

It is exactly the “social contract” between demonstrator and the public—“each step in the derivation has the appropriate ideas corresponding to it”—that was violated by the “Great Author and his followers” in the field of analysis, thus bringing on a crisis of reason severe enough for Berkeley to return to it “after so long an Intermission of these Studies” (Analyst 50). This return was especially imperative in a period when ideas themselves were losing their exclusive role in Berkeley’s ontology.

24 In my exposition of Berkeley’s views on geometry and the calculus I will refrain from pointing out a number of important problems that Berkeley’s philosophy of geometry posed even when applied to the most ideal setting. For more on these problems, see Brook, Berkeley’s Philosophy of Science, 164-69.
A similar crisis of representation was taking place in the money form throughout Europe and its colonies, but in Ireland especially. The millennia-old money system based on gold and silver coinage (which set the stage, according to Sohn-Rethel and others, for the development of the abstraction necessary for geometry) was in crisis in the Ireland Berkeley found on returning to claim his bishopric in 1734. Simply put, Ireland was experiencing a monetary catastrophe that was described vividly by Berkeley’s friend, lawyer and political confidant, Thomas Prior, who in 1729 “calculated that in contemporary England there was available forty shillings per head of population, 13s, 4d in silver and the rest in gold, whereas in Ireland there was only 4s, 5.25d per head, of which 5d was in silver and the rest in gold.”

This ten-to-one ratio of specie in England and Ireland deeply worried Berkeley, Prior, and their circle, since it seemed to condemn Ireland to perpetual poverty. On their analysis, the gold and silver money supply was not adequate for the needs of the country, for practical and theoretical reasons. In practice, they argued, (i) gold and silver coins made it possible for absentee landlords to live abroad and to have their rents (paid in gold or silver money) transported immediately to London or the continent, and (ii) the denominations of gold and silver coins were too large to be used for the small change required by rural cottiers and urban workers in their markets.

Theoretically, Berkeley and his friends refused to accept the mercantilist identification of the quantity of specie within a country with its wealth; and they refused to believe that the referent of a unit of money was an intrinsic value measured by an amount of precious metal. They maintained that the best measure of national wealth is coordinated collective activity (what Berkeley called “the momentum of the State”), which would increase every individual man’s (and woman’s?) power “according to his just pretensions and industry.” In Ireland’s case, though, gold and silver coinage was not conducive to increasing the momentum of the state. Moreover, the obsession to find and fix the correct referent of money as a value reflected by the gold or silver in a coin is misguided and destructive in a poor country like Ireland where disenfranchised “native” workers have developed a “cynical content” and were unwilling to exert themselves. This monetary semantics undermined the true function of money (which is notional), for the whole point of money is not to refer to a sort of thing or a collection of ideas (in the way a word like “stone” refers to a thing). It is rather to promote, transfer, and secure a commerce in the command over human labor—all of which are quite notional entities.

III.b The Critique of the Certainty of Self-Reflexive Representation

Where, according to Berkeley, did the two great systems of representation in mathematics and money go wrong? I believe that he traced the source of their crises (though not the motivations behind them) in their similar virtues—the chief being the self-reflexive aspect of their signifiers: diagrams in geometry, and precious metal coins for money. Both diagrams and specie coinage had their truth literally “written in them.” Purportedly, they have a self-correcting, self-evident feature that the rival forms of representation (i.e.,

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algebra and paper currency) do not; hence their superiority. In particular, *The Analyst* and *The Querist* question, respectively, the incorruptible epistemological virtues of geometry and precious metal coinage.

For example, at the beginning of *The Analyst*, Berkeley sets out the geometric code of behavior:

> It hath been an old remark that Geometry is an excellent Logic. And it must be owned, that when the Definitions are clear; when the Postulata cannot be refused, nor the Axioms denied; when from the distinct Contemplation and Comparison of Figures, their Properties are derived, by a perpetual well-connected chain of Consequences, the Objects being still kept in view, and the attention ever fixed upon them; there is acquired a habit of reasoning, close and exact and methodical: which habit strengthens and sharpens the Mind, and being transferred to other Subjects, is of general use in the inquiry after Truth. But how far this is the case of our Geometrical Analysts, it may be worth while to consider. (*Analyst* 2)

The most important point about this “old remark” for us is the emphasis on “the distinct Contemplation and Comparison of Figures . . . the Objects being still kept in view, and the attention ever fixed upon them.” What is crucial about geometry is not only its deductive character, but its valuation of figures (geometric diagrams) in the process of deduction. Any student of geometry would understand Berkeley’s point, for the whole charm of geometric truth is its almost hallucinatory connection with the diagram. Truth seems to grow out of it, while it exemplifies this truth in its being.

Berkeley saw the diagram as an essential part of the process of geometric deduction from the beginning of his philosophical work. For example, in his description of how a geometric proof that deals with one particular triangle can be generalized, he writes in the Introduction of the *Principles*:

> though the idea I have in view whilst I make the demonstration, be, for instance, that of an isosceles rectangular triangle, whose sides are of determinate length, I may nevertheless be certain it extends to all other rectilinear triangles, of what sort of bigness soever. And that, because neither the right angle, nor the equality, nor determinate length of the sides, are at all concerned in the demonstration. It is true, the diagram I have in view includes all these particulars, but then there is not the least mention made of them in the proof of the proposition. (PHK IN 16)

In this analysis of generalization in geometry, the diagram plays an essential role in setting up the problem.\(^{26}\) Indeed, in the case at hand, it is a particular idea that is to be generalized and be a sign of all triangles. Nonetheless, the proper functioning of the diagram in geometric reasoning requires that a sort of social contract be made between the demonstrator and the public. That is, nothing can be brought into the demonstration that is not “in view.” The proof must be clear, transparent and open to generalized

\(^{26}\) On the interpretational problems posed by Berkeley’s insistence on the importance of diagrams in geometric reasoning, see Brook, *Berkeley’s Philosophy of Science*, 164-68.
attention, for the issue of generalization applies not only with respect the object, but also with respect to the general attention that is being paid to the demonstration by the mathematical public.

Berkeley clearly expresses this requirement in Section 34 of *The Analyst* where he writes:

> If it is said that Fluxions may be expounded or expressed by finite lines proportional to them: Which finite Lines, as they may be distinctly conceived and known and reasoned upon, so they may be substituted for the Fluxions, and their mutual Relations or Proportions be considered as the Proportions of Fluxions: By which means the Doctrine becomes clear and useful. I answer that if, in order to arrive at these finite Lines proportional to the Fluxions, there be certain Steps made use of which are obscure and inconceivable, be those finite lines themselves ever so clearly conceived, it must nevertheless be acknowledged that your proceeding is not clear nor your method scientific.

In the dominant representational system of money, that is, specie, the power of that system is dependent on its self-reflexive character. In its terms, something is a sign of value V precisely because it has an “intrinsic” value V. It can take a “natural” place in the realm of exchange as something of a god, since it can measure the value of all other commodities through its own self-evident value. Berkeley questions the power of this self-reflexive character of specie by subverting the traditional conception of money when he asks “Whether money is to be considered as having intrinsic value, or as being a commodity, a standard, a measure, or a pledge, as is variously suggested by writers?” (Querist 23). His negative answer to this question is suggested in the remaining part of the same query: “And whether the true idea of money, as such, be not altogether that of a ticket or counter?”
In fact, throughout The Querist Berkeley dethrones the self-reflexive semantic role of specie and promotes another role that makes it nugatory. For example, consider the following queries (employing the standard ethnic prejudices of the day):

What makes a wealthy people? Whether mines of gold and silver are capable of doing this? And whether the negroes, amidst the gold sands of Afric, are not poor and destitute? (Querist 29)

Whether there be any virtue in gold or silver, other than as they set people at work, or create industry? (Querist 30)

Whether even gold or silver, if they should lessen the industry of its inhabitants, would not be ruinous to a country? And whether Spain be not an instance of this? (Querist 43)

In other words, the crucial semantic questions that are posed by those who take the intrinsic value of gold and silver—for example, what is the ratio of fine gold to fine silver in various European countries?—are not relevant to Berkeley’s concept of money. Newton, as Master of the Mint, wrote concerning this ratio in his Memorial on the State of the Gold and Silver Coin (1717):

In the end of King William’s reign, and the first year of the late queen [Anne], when foreign coins abounded in England, I caused a great many of them to be assayed in the mint, and found by the assays that fine gold was to fine silver in Spain, Portugal, France, Holland, Italy, Germany, and the northern kingdoms.27

His research led him to recommend that the gold guinea be reduced by 6d to a value of 21s in order to increase the silver coinage in Britain. The result, however, was not as Newton expected: “After 1717 less than £600,000 worth of silver was minted during the rest of the [18th] century, while for the same period well over £70 million of gold coin was produced.”28 Thus Newton inadvertently presided over the initiation of the gold standard that was imposed worldwide with the triumph of Britain in the Napoleonic wars a century later.

Berkeley’s response to such investigations and recommendations was to show that the purported self-reflexive intrinsic value of specie did not have the transcendent virtues attributed to it. He queried the Newtons of the world who took the gold/silver exchange rate as the crucial monetary variable: “Whether altering the proportions between the several sorts can have any other effect but multiplying one kind and lessening another without increase of the sum total?” (Querist 27). In other words, the very variability of the ratio of exchange between gold and silver indicated that the claimed god-like objective status was questionable.

27 Reprinted in Considerations on the silver currency . . . containing a report of Sir Isaac Newton on the state of the gold and silver coin (Dublin: J. Miliken, 1805), 49.
Indeed, what Berkeley did was simply to change the goal of the semantic game of money by substituting, for questions like “How much gold was in this guinea?” to questions like “How much industry does this guinea excite?” He thus literally changed the subject.

III.c The Revaluation of Systems of Representation

_The Analyst_ and _The Querist_ revealed crises in forms of representation in money and mathematics that had been dominant for more than two millennia. Ironically, in the moment of triumph and expansion of both the geometric and the metallist forms of representation in the 18th century, Berkeley found a rapidly disintegrating situation that required an intellectual and semantic revolution: new forms of representation had to replace the old, failing ones. Inevitably, he turned to the “other” representational systems in their respective fields—algebra and paper currency—and revalued them.

In the last section of _The Analyst_ Berkeley puts forth a number of queries that ask the reader to revalue algebra. In his previous writings, he had had very little to say about algebra, and what little he said was presented in a light ludic manner. Indeed, his first publication _Miscellanea Mathematica_ in 1707 included “De Ludo Algebraico,” a text that literally transformed algebra into a game. It is a description of an algebraic board game Berkeley invented “for the entertainment of undergraduates as a way to randomly generate a set of algebraic equations (or questions in the parlance of the day) that had to be solved competitively.”  

29 The players randomly generated a series of algebraic equations (or “questions”) and competed in solving them.

He wrote of algebra then, “You see what a mere game algebra is, and that both chance and science have a place in it. Why not, therefore, come to the gaming table?”  

30 It was an important game, for he did give algebra fulsome praise:

> And, indeed, how difficult would it be to assign the limits of algebra, when it has latterly extended to natural philosophy and medicine, and daily sets about the most valuable arguments. . . . it may be laid down for certain that wherever greater and less are brought forward, wherever any ratio or proportion can be admitted, there algebra finds a place.  

31

In his few other direct references to algebra, he uses it as an example to make a point about how symbolic systems do not satisfy the Lockean semantic program that requires that one sign = one idea.  

32 Thus in the _Principles_ he writes of algebra as a system “in which though a particular quantity be marked by each letter, yet to proceed right it is not requisite that in every step each letter suggest to your thoughts, that particular quantity it

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29 Caffentzis, _Exciting the Industry_, 262.
30 Translated in Jesseph, _Berkeley's Philosophy of Mathematics_, 115.
32 Caffentzis, _Exciting the Industry_, 188-94.
was appointed to stand for” (PHK IN 19). And in *Alciphron* VII.14 he implicates algebra with the realm of notions.³³ For he begins the section by declaring that:

\[
\text{signs . . . have other uses besides barely standing for and exhibiting ideas, such as raising proper emotions, producing certain dispositions or habits of mind, and directing our actions in pursuit of that happiness, which is the ultimate end and design, the primary spring and motive, that sets rational agents at work: that signs may imply or suggest the relations of things; which relations, habitudes or proportions, as they cannot be by us understood but by the help of signs, so being thereby expressed and confuted, they direct an enable us to act with regard to things.}
\]

Clearly, emotions (e.g., love and hate), dispositions or habits of mind, directions of action, and relations are exactly what Berkeley defined as notions in the 1734 revision of the *Principles* and the *Dialogues*. He points out in the notional realm there can be “a conceived good” even though it cannot be exhibited as an idea to the mind. He then turns, as clinching example, to what he calls an algebraic sign, the square root of a negative number, and claims that it is useful “in logistic operations, although it be impossible to form an idea of any such quantity” (*Alciphron* VII.14).

Another way of seeing the relation of algebra to the notion is by noting its closeness to arithmetic in Berkeley’s thought. As Jesseph puts it, Berkeley’s philosophy of arithmetic “extends without significant modification to include algebra.”³⁴ This connection is not unique to Berkeley, of course. Many commercial arithmetic texts follow their sections on the mechanics of addition, subtraction, multiplication and division with a set of problems from the business world that we would now consider algebraic (e.g., the “rule of three” in the *Treviso Arithmetic* of 1478).³⁵ But Berkeley’s view of arithmetic (and thus algebra) is both formalist and instrumentalist. As he writes in the *Principles*: “In arithmetic therefore we regard not the things but the signs, which nevertheless are not regarded for their own sake, but because they direct us how to act with relation to things and dispose rightly of them” (PHK 122). Berkeley’s emphasis on action with respect to arithmetic (and algebra) is evident and immediately implicates them with a notional function in mental economy. Arithmetic and algebra (unlike geometry) direct the “how,” but they do not demonstrate the “that.” They involve the “right” and “wrong” disposition of the will, not the “true” or “false” of the understanding. But as Flage has argued, the former is exactly what notions are, “the actions of the mind or disposition of the mind to act in certain ways.”³⁶ Thus arithmetic and algebra are directly notional while geometry remains ideational.

Given the difficulties of a geometry embroiled with fluxions and the gradually improving status of algebra in his thought, it should not be surprising that by 1734 Berkeley would have raised algebra to the status of a science. Consider the series of queries concerning algebra in *The Analyst* Qu. 41–46, including:

³³ Citations of *Alciphron* (by dialogue number and section) are taken from *Alciphron in Focus*, ed. David Berman (London: Routledge, 1993).
³⁴ *Berkeley’s Philosophy of Mathematics*, 284.
³⁶ Flage, *Berkeley’s Doctrine of Notions*, 188.
Whether in the most general Reasonings about Equalities and Proportions, Men may not demonstrate as well as in Geometry? Whether in such Demonstrations, they are not obliged to the same strict Reasoning as in Geometry? And whether such their Reasonings are not deduced from the same Axioms with those in Geometry? Whether therefore Algebra be not as truly a Science as Geometry? (Analyst Qu. 41)

Whether, although Algebraic Reasonings are admitted to be ever so just, when confined to Signs or Species as general Representatives of Quantity, you may not nevertheless fall into Error, if, when you limit them to stand for particular things, you do not limit your self to reason consistently with the Nature of such particular things? And whether such Error ought to be imputed to pure Algebra? (Analyst Qu. 46)

This transformation of Algebra into a science that can include notions (that is, significant, useful, but non-idealational elements like “imaginary numbers”) is a decisive development in Berkeley’s philosophy of mathematics. Paradoxically, indeed, Berkeley’s move makes possible an even higher level of abstraction in mathematics than afforded by those fields that were still in the thrall of the diagram.

A similar transformation takes place for paper currency in The Querist. It should be clear that paper currency was an alternative to specie for some time before 1734, and it joined with a set of other paper instruments like bills of exchange, stocks, checks, and debt documents to overwhelm gold and silver coinage in Europe. As Fernand Braudel points out about 18th century European economies:

In Amsterdam, London and Paris, we have seen that company shares were quoted on the Exchanges. Add to this “bank notes” of various origin and one has an enormous mass of paper money. Sages at the time said that it should not be more than three or four times the value of the mass of metal money. But ratios of 1 to 15 or more are extremely probable at certain periods in Holland and England.\(^37\)

Moreover, Berkeley had direct experience living in a largely specie-less economy in Rhode Island a few years before.\(^38\) Consequently, the proposal to “de-specie-ize” the Irish economy was not as unprecedented as it might have sounded.

It is true that paper currency was being criticized as dangerous throughout the Atlantic world in the aftermath of Law’s experiment in 1719-20 and the inflation in the North American colonies presumably brought about by unregulated paper money creation. But though he recognized the difficulties posed by paper currency, Berkeley was convinced that if it were properly regulated, paper currency in Ireland would solve many social difficulties. Money in Berkeley’s view was notional, in that it did not represent an idea or a collection of ideas but rather was involved in “raising proper emotions, producing certain dispositions or habits of mind, and directing our actions in pursuit of that


\(^{38}\) Caffentzis, Exciting the Industry, 80-100.
happiness, which is the ultimate end and design [of human activity]” (*Alciphron* VII.14). But paper money is exponentially notional in that it does not represent any particular value, but in Berkeley’s scheme it is literally created by a National Bank. His proposal was that a National Bank be founded, “That Bank Notes be minted (a) to the Value of one hundred thousand Pounds, in round numbers for one Pound to Twenty. (b) That such Notes be issued, either to particular Persons on Cash or Security; or else, to the Uses of the Publick on its own Securities.” Indeed, this Bank need not have any starting “fund” at all, and hence not even need a “fig leaf” of representationality (although Berkeley was not against providing such a “fig leaf” if it was politically required.

Ironically enough, if the notional aim of the paper currency (viz., exciting industry) succeeds, then it will eventually lead to a healthy economy and “in the event, multiply our Gold and Silver.” So Berkeley urges, in order to increase the skill and industry of the people, they must be encouraged by “ready Payments” (as the children Berkeley was referring to in this essay’s epigraph):

> These Payments must be made with Money, and Money is of two sorts: Specie or Paper. Of the former, we neither have a sufficient Quantity, nor yet Means of acquiring it. Of the latter Sort, we may have what we want, as good and current as any Gold for Domestic Uses. Why should we therefore reach forth our Hand, and take of that Sort of Money which is in our Power; and which makes far the greater Part of the Wealth of the most flourishing States in Europe?

Berkeley’s revaluation of paper money and his emphasis on the notional, non-ideational aspects of money were simultaneous, interacting developments. Indeed, I claim that his philosophical revaluation of notions (as well as principles, opinion and even prejudices) gave him the intellectual ability to challenge the powers supporting specie, while his philosophy of money concretized his “second conceptual revolution.”

### III.d Algebraic Money: The Importance of the Ludic (Counters)

I have traced parallel developments in the dual systems of representation in mathematics and money in Berkeley’s thought. Now the question is: do the parallels ever intersect? Is there a textual support for going beyond mere homology to actually connecting algebra with paper currency?

I believe that there is, if we consider the notion of money as a counter. Indeed, Berkeley asks in *Querist* 23 “Whether the true Idea of Money, as such, be not altogether that of a Ticket or Counter?” But what does the counter count? He immediately suggests the answer: “Whether the value or price of things be not a compounded proportion, directly as the demand, and reciprocally as the plenty?” (*Querist* 24); and “Whether the terms

40 Caffentzis, *Exciting the Industry*, 293-94.
crown, livre, pound sterling, etc. are not be considered as exponents or denominations of such proportion? And whether gold, silver, and paper are not tickets or counters for reckoning, recording and transferring thereof?” (Querist 25).

According to the OED, in early 18th century mathematical terminology, an “exponent” is “the ratio or proportion between two numbers or quantities, the quotient arising when the antecedent is divided by the consequent. Thus 6 is the exponent of the ratio that 30 was to 5.” Counters thus count ratios and proportions. But in Berkeley’s 1734 terms, ratios and proportions are relations and therefore notions. Because “general reasonings about equalities and proportions” are identified with algebra (Analyst Qu. 41), the counters that reckon, record, and transfer these proportions are doing algebra-like operations.

The connection between money, algebra, and the queries found in the works of the mid-1730s thus retrieves a theme that Berkeley had raised as early as his “De Ludo Algebraico.” As I noted in Exciting the Industry of Mankind:

the mature Querist poses “questions” just as the youthful player of the Algebraic Game finds chance has posed for him or her “questions” in the form of algebraic equations. The Querist’s solution is to be found in the algebraic movement of the spirits which have been released from the delusion that their pegs and counters are the “solution” to their question, rather it is their activity that is the solution. The Irish economy has to be ludified if the solution to the final question of The Querist, was to be found, according to the Querist.42

For Berkeley, “algebraic money” was the solution to the fundamental economic problems of Ireland. Accordingly, the primacy of the question as the linguistic correlate for money is vindicated and the parallel crises are brought together.

Conclusion

Berkeley was a subtle thinker, especially later in his life. Even though he was always a passionate advocate throughout his life, the brash but powerful dichotomies of his youth were replaced by a more nuanced conception of the subjects of his studies, including mathematics and money. His promotion of algebra and paper currency over geometry and specie was not eliminative. On the contrary, he was anxious in both the fields of mathematics and money to devise more generalized and effective notions of rigor that would harmonize the past (i.e., Euclidean geometric reasoning and the specie-dominated monetary system) with the needs of the present (i.e., a coherent presentation of the powerful results of the calculus and a recognition of the increasing importance of paper and credit-based monetary instruments). “Algebraic money” is therefore a phrase that

42 Caffentzis, Exciting the Industry, 262-63.
tries to capture Berkeley’s effort to achieve this vital harmony of the past and the new in two major fields of representation: mathematics and money.\footnote{Berkeley’s \textit{Analyst} had an impact on the more self-conscious development and differentiation of an algebraic and a geometric approach to the calculus. Colin Maclaurin’s influential geometric book on the calculus, \textit{Theory of Fluxions} [1742], was partly written as a response to Berkeley’s criticism of Newton’s theory, while Louis Lagrange’s work on the calculus at the end of the 18th century seemed to agree with Berkeley’s suspicion of the ethics of the fluxion diagram-makers [see Carl B. Boyer and Uta C. Merzbach, \textit{A History of Mathematics}, 2nd ed. (New York: John Wiley and Son, 1989), 480]. As Judith V. Grabiner writes, “For Maclaurin, the calculus was at heart geometric; for Lagrange, the calculus was algebraic. Maclaurin’s great \textit{Treatise of Fluxions} has over 350 diagrams; Lagrange’s masterwork on the calculus, the \textit{Théorie des functions analytiques}, search as one will, contains none—just pages of text and formulas” [“The Calculus as Algebra, the Calculus as Geometry: Lagrange, Maclaurin and their Legacy,” in \textit{Vita Mathematica: Historical Research and Integration with Teaching}, ed. Ronald Calinger (Washington, D.C.: Mathematical Association of America, 1996), 132].}