1. (a) Find $f'(x) = 3x^2 + 6x - 9$. Set it equal to 0 and solve for $x$. Get $x = -3$ and $x = 1$. Choose test points, say $x = -4, 0,$ and $2$. Compute that $f'(-4) > 0$, $f'(0) < 0$, and $f'(2) > 0$. So $f$ is increasing on $(-\infty, -3)$ and $(1, \infty)$ and $f$ is decreasing on $(-3, 1)$.

(b) Based on part (a), a relative max of $f$ occurs at $x = -3$ and a relative min occurs at $x = 1$. The relative max is $f(-3) = 0$ and the relative min is $f(1) = -32$.

(c) Find $f''(x) = 6x + 6$. Set it equal to 0 and solve for $x$. Get $x = -1$. Choose test points, say $x = -2$ and $0$. Compute that $f''(-2) < 0$ and $f''(0) > 0$. So $f$ is concave down on $(-\infty, -1)$ and concave up on $(-1, \infty)$.

(d) Based on part (c), an inflection point of $f$ occurs at $x = -1$. The inflection point is $(-1, -16)$.

2. Your sketch should look something like this:

![Sketch of the function](1.jpg)

3. Since the numerator and denominator have the same degree, the limit is the ratio of their leading coefficients, namely $5/2$.

4. Find $f'(x) = 3 - 10000/x^2$. Set it equal to 0 and solve for $x$. Get $x = 100/\sqrt{3} = 57.735$. Use the first derivative test to confirm this. Use test points, say $x = 50$ and $100$. Compute that $f'(50) = 3 - 4 < 0$ and $f'(100) = 3 - 1 > 0$. Thus, a min occurs at $x = 57.735$. That is the width. The length is $5000/57.735 = 86.6$, since the area must be 5000.

5. The function to use is $f(x) = x^3 - 10$. So $f'(x) = 3x^2$. Then the formula is

$$x_{n+1} = x_n - \frac{x_n^3 - 10}{3x_n^2}.$$  

If $x_1 = 2$, then $x_2 = 2.166667$ and $x_3 = 2.147633745$.

6. According to the Pythagorean theorem, $r^2 + \left(\frac{h}{2}\right)^2 = R^2$. So $r = \sqrt{R^2 - \frac{1}{4}h^2}$. Substitute this into the formula for volume:

$$V = \pi r^2 h = \pi (R^2 - \frac{1}{4}h^2) h = \pi R^2 h - \frac{\pi h^3}{4}.$$
Find the derivative: \( V' = \pi R^2 - \frac{3\pi}{4} h^2 \). Set this equal to 0 and solve for \( h \). Get \( h = \frac{2R}{\sqrt{3}} \). For the second derivative test, find \( V'' = -\frac{3\pi}{2} h \). Evaluate this at \( h = \frac{2R}{\sqrt{3}} \) and get a negative result, indicating a max.

7. In the equation \( h = b \tan \theta \), let \( b = 100 \). Then differentiate the equation to get

\[
dh = 100 \sec^2 \theta d\theta.
\]

Let \( d\theta \) be 1°, expressed in radians (\( 1^\circ = \frac{\pi}{180} \) radians). Let \( \theta \) be 30°. Substitute and compute the value of \( dh \). Get \( dh = 2.327 \).