1. Write
\[ \sum_{n=1}^{5} \frac{(-1)^{n+1}2^{n-1}}{n+1}. \]
This could be simplified to
\[ \sum_{n=1}^{5} \frac{(-2)^{n-1}}{n+1}, \]
or
\[ \sum_{n=0}^{4} \frac{(-2)^n}{n+2}. \]

2. The simplest method is to use algebra:
\[
\sum_{k=1}^{n} (2k + 3) = \sum_{k=1}^{n} 2k + \sum_{k=1}^{n} 3
\]
\[
= 2 \left( \frac{n(n+1)}{2} \right) + 3n
\]
\[
= n(n+1) + 3n
\]
\[
= n^2 + 4n.
\]

Another method would be to find a formula for the sum of the odd integers (we did that in class), namely,
\[
\sum_{k=1}^{n} (2k - 1) = n^2.
\]

Then apply this to the given sum:
\[
\sum_{k=1}^{n} (2k + 3) = \sum_{k=3}^{n+2} (2k - 3)
\]
\[
= \sum_{k=1}^{n+2} (2k - 3) - (1 + 3)
\]
\[
= (n + 2)^2 - 4
\]
\[
= n^2 + 4n.
\]

3. When \( n = 1 \), \( \sum_{k=1}^{n} k(k+1) = 1 \cdot 2 \) and \( \frac{n(n+1)(n+2)}{3} = \frac{1 \cdot 2 \cdot 3}{3} = 2 \). Therefore, the statement is true when \( n = 1 \).
Now suppose that the statement is true when \( n = m \), for some integer \( m \geq 1 \). We will show that it is true when \( n = m + 1 \).

\[
\sum_{k=1}^{m+1} k(k + 1) = \sum_{k=1}^{m} k(k + 1) + (m + 1)(m + 2)
\]
\[
= \frac{m(m+1)(m+2)}{3} + \frac{3(m+1)(m+2)}{3}
\]
\[
= \frac{(m+1)(m+2)(m+3)}{3}
\]

Therefore, the statement is true for all \( n \geq 1 \).

4. Let \( n = 1 \). Then \( a_n = a_1 = 1 \) and \( 3 \cdot 2^1 - 5 = 3 \cdot 2^1 - 5 = 1 \). Therefore, the statement is true when \( n = 1 \).

Now suppose that the statement is true when \( n = k \), for some integer \( k \geq 1 \). We will show that the statement is true when \( n = k + 1 \).

\[
a_{n+1} = 2a_n + 5
\]
\[
= 2(3 \cdot 2^n - 5) + 5
\]
\[
= (3 \cdot 2^{n+1} - 10) + 5
\]
\[
= 3 \cdot 2^{n+1} - 5.
\]

Therefore, the statement is true for all \( n \geq 1 \).

5. Let \( A = \{1, 2, 4, 8\} \), \( B = \{1, 3, 5, 7\} \), and \( C = \{2, 3, 5, 7\} \) and let the universal set be \( \{1, 2, 3, 4, 5, 6, 7, 8\} \). Then

(a) \( A \cup B = \{1, 2, 3, 4, 5, 7, 8\} \)

(b) \( B \cap C = \{3, 5, 7\} \)

(c) \( (A \cup B) - (A \cap B) = \{2, 3, 4, 5, 7, 8\} \)

(d) \( (B \cap C)^c = \{1, 2, 4, 6, 8\} \)

6. Suppose such a program \( \text{ZERO} \) exists. Then use it to build a program \( \text{TEST} \) as follows:

The program \( \text{TEST} \) will read a program \( P \). It will then run \( \text{ZERO} \) with input \( P \) (as the program) and \( P \) (as the input to \( P \)). \( \text{ZERO} \) will determine whether \( P \)
will output 0 on input \( P \). If \( \text{ZERO} \) reports “yes,” then \( \text{TEST} \) will output 1, and if \( \text{ZERO} \) reports “no,” then \( \text{TEST} \) will output 0. Now run \( \text{TEST} \) on input \( \text{TEST} \) and we will have a contradiction. If \( \text{TEST} \) should output 0 on input \( \text{TEST} \), then \( \text{ZERO} \) will report “yes,” so \( \text{TEST} \) will output 1. And if \( \text{TEST} \) should not output 0 on input \( \text{TEST} \), then \( \text{ZERO} \) will report “no,” so \( \text{TEST} \) will output 0.

7. (a) There are six possible choices:

\[ \{ R_1R_2, R_1B_1, R_1B_2, R_2B_1, R_2B_2, B_1B_2 \}. \]

(b) Of the six equally likely choices in part (a), 2 of them are of the same color \( \{ R_1R_2, B_1B_2 \} \). Therefore, the probability is \( \frac{2}{6} = \frac{1}{3} \).

8. (a) There are \( \left\lfloor \frac{10000}{29} \right\rfloor = 344 \) multiples of 29, \( \left\lfloor \frac{10000}{37} \right\rfloor = 270 \) multiples of 37, and \( \left\lfloor \frac{10000}{89} \right\rfloor = 112 \) multiples of 89. Furthermore, there are \( \left\lfloor \frac{29 \cdot 37 \cdot 89}{10000} \right\rfloor = 9 \) multiples of 29 \( \cdot \) 37, \( \left\lfloor \frac{29 \cdot 37}{10000} \right\rfloor = 3 \) multiples of 29 \( \cdot \) 89, and \( \left\lfloor \frac{37 \cdot 89}{10000} \right\rfloor = 3 \) multiples of 37 \( \cdot \) 89. There are \( \left\lfloor \frac{29 \cdot 37 \cdot 89}{10000} \right\rfloor = 0 \) multiples of 29 \( \cdot \) 37 \( \cdot \) 89. Using the Inclusion-Exclusion Principle, the number of numbers from 1 to 10000 that are multiples of 29, 37, or 89 is

\[ 344 + 270 + 112 - 9 - 3 - 3 + 0 = 711. \]

(b) If 711 of the numbers are multiples of 29, 37, or 89, then 10000 – 711 = 9289 of the numbers are not multiples of 29, 37, or 89. If we choose one at random, the probability is \( \frac{9289}{10000} = 0.9289 \) that it is not a multiple of 29, 37, or 89.

9. The total number of ways to choose any 3 marbles from the 10 is \( \binom{10}{3} = 120 \). To choose 3 marbles of the same color is to choose 3 red marbles and 0 green marbles or 0 red marbles and 3 green marbles. The number of ways to do this is \( \binom{5}{3} \binom{5}{0} + \binom{5}{0} \binom{5}{3} = 10 \cdot 1 + 1 \cdot 10 = 20 \). So the probability is \( \frac{20}{120} = \frac{1}{6} \).

10. Expand as

\[ (a + 2b)^5 = a^5 + \binom{5}{1}a^4(2b) + \binom{5}{2}a^3(2b)^2 + \binom{5}{3}a^2(2b)^3 + \binom{5}{4}a(2b)^4 + (2b)^5 \]

\[ = a^5 + 10a^4b + 40a^3b^2 + 80a^2b^3 + 80ab^4 + 32b^5. \]