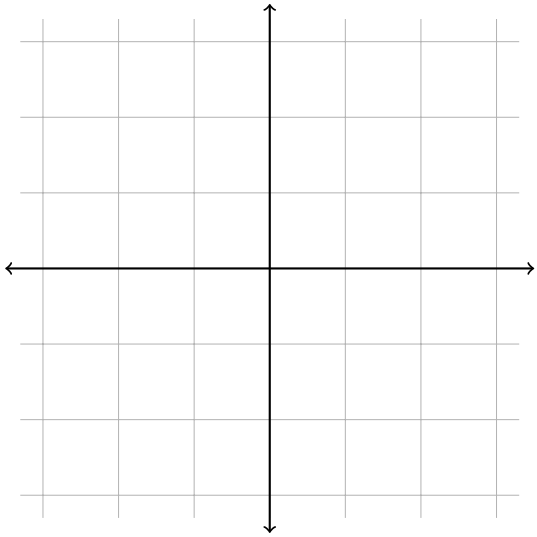
Vector Geometry I **Math 231 - Workshop**

A *line* in $\mathbb{R}^n$ is the set of all vectors of the form

$$\mathbf{x} + t\mathbf{y},$$

where $\mathbf{x}, \mathbf{y}$ are fixed vectors in $\mathbb{R}^n$ ($\mathbf{y} \neq \mathbf{0}$) and $t \in \mathbb{R}$ is allowed to vary. The formula $\mathbf{x} + t\mathbf{y}$ is called the *parametric form* of the line.

1. Draw a picture of the line $\begin{bmatrix} 3 \\ -2 \end{bmatrix} + t \begin{bmatrix} -1 \\ 1 \end{bmatrix}$. Does the point $\begin{bmatrix} 0 \\ 1 \end{bmatrix}$ lie on this line? If so, what is the value of t at that point?



To find the parametric form of a line,

- $\mathbf{x}$ can be any point on the line, and
- $\mathbf{y}$ can be any vector that points in the direction of the line.

A line is completely described by a *point* $\mathbf{x}$ and a *direction* $\mathbf{y}$.

2. Find an expression in parametric form for the line in $\mathbb{R}^2$ that passes through $\mathbf{u} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$ and $\mathbf{v} = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$.

3. Do the three vectors

$$\mathbf{u} = \begin{bmatrix} 1 \\ 2 \\ 0 \\ 3 \end{bmatrix}, \mathbf{v} = \begin{bmatrix} 3 \\ 2 \\ -2 \\ -1 \end{bmatrix}, \text{ and } \mathbf{w} = \begin{bmatrix} 0 \\ 2 \\ 1 \\ 5 \end{bmatrix},$$

all lie on the same line? How can you tell?

4. Do the two lines below intersect? If so, where? If not, explain how you know.

$$\left\{ \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix} s \ : \ s \in \mathbb{R} \right\} \text{ and } \left\{ \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix} + \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix} t \ : \ t \in \mathbb{R} \right\}$$

5. Does the line

$$\left\{ \begin{bmatrix} 7 \\ 0 \\ 0 \end{bmatrix} + r \begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix} : r \in \mathbb{R} \right\}$$

intersect the plane

$$\left\{ \begin{bmatrix} 0 \\ 7 \\ 9 \end{bmatrix} + s \begin{bmatrix} 5 \\ -4 \\ -1 \end{bmatrix} + t \begin{bmatrix} 2 \\ -1 \\ 3 \end{bmatrix} : s, t \in \mathbb{R} \right\}?$$

If so, where?