Basic Probability Rules **Workshop**

An **event** is a subset of the possible outcomes in a probability model. We use capital letters like A or E to represent events, and $P(E)$ is short-hand for the phrase *"the probability that event E happens"*.

Two events are **mutually exclusive** if they cannot both happen at the same time.

Two events are **independent** if the probability of one doesn't change when the other happens.

Addition Rule: If A and B are mutually exclusive, then

$$P(A \text{ or } B) = P(A) + P(B).$$

Multiplication Rule: If A and B are independent, then

$$P(A \text{ and } B) = P(A) \cdot P(B).$$

1. For each of the following pairs of events, decide whether they are independent or not.
   (a) It rains today *and* the baseball game today is canceled.

   (b) You win the lottery *and* it rains next week.

   (c) A random person was a cheerleader in high school *and* they are female.

2. Bob is taking a multiple choice test. Each question has five options. For the last two questions, Bob has no clue which answer is correct, so he guesses.
   (a) What is the probability that Bob gets both questions wrong?

   (b) What is the probability that Bob gets both questions right?

   (c) What is the probability that Bob gets one question wrong and one question right? Hint: You've already found the probabilities that he gets both right and both wrong. What's left?

A **conditional probability** is the probability of an event A happening if you know that an event B has happened. We use the short-hand $P(A|B)$ to represent *"the probability of A given B"*.

General Multiplication Rule: If A and B are any events (independent or not), then

$$P(A \text{ and } B) = P(A) \cdot P(B|A).$$

3. It is estimated that women in their 40s have a 0.8% chance of having breast cancer. Mammograms are 90% accurate at detecting breast cancer, if a woman has it. Mammograms are 93% accurate if a women does not have breast cancer.

   (a) What is $P(\text{Test positive} \mid \text{Does not have breast cancer})$?

   (b) The number 90% is a conditional probability, so it can be expressed as $P(A|B)$. Describe in words what the events A and B are.

   (c) For women in their 40s who get mammograms, what percent test positive and have breast cancer?

   (d) For women in their 40s who get mammograms, what percent get false positives (they test positive, but don't have breast cancer)?

   (e) Find the conditional probability that a woman actually has breast cancer if she gets a positive mammogram result by using the formula

$$P(A|B) = \frac{P(A \text{ and } B)}{P(B)}.$$