Algebraic Structures Homework #4
Due Friday, November 14

Although this homework is due November 14, you will have a chance to revise your answers after I return it to you on Monday, November 17.

(1) Let H be a normal subgroup in G and suppose that $[G : H] = m$. Prove that for every $g \in G$, $g^m \in H$. (Hint: You will probably need to use the fundamental homomorphism theorem.)

(2) Show that if a finite group G has a subgroup H, then for any $g \in G$, the set $g^{-1}Hg$ is also a subgroup of G, and the order of $g^{-1}Hg$ is the same as the order of H.

(3) Use the conclusion of problem #2 to prove that if H is the only subgroup of a finite group G with a given order, then H is normal.

(4) In a ring R, we say that a *divides* b and write $a \mid b$, if there exists an element c in R such that $b = ac$.
(a) Show that $4 \mid 2$ in $\mathbb{Z}_6$.
(b) Show that $3 \mid 7$ in $\mathbb{Z}_8$.

(5) (Extra Credit) A Boolean ring is a ring R with the property that $a^2 = a$ for all $a \in R$. Show that any Boolean ring is commutative.