Algebra Review Quiz Solutions

Simplify the given expressions.

(1) $\dfrac{1}{x+1} + \dfrac{1}{x-1}$

In order to add two fractions, they need to have a common denominator. (**Remember that the denominator is the bottom of the fraction and the numerator is the top.**) So, I multiply the first fraction by $(x-1)$ on the top and bottom. Similarly, I multiply the second fraction by $(x+1)$ on the top and bottom.

$$= \frac{(x-1)}{(x-1)}\frac{1}{x+1} + \frac{1}{x-1}\frac{(x+1)}{(x+1)}$$

$$= \frac{x-1}{(x-1)(x+1)} + \frac{x+1}{(x-1)(x+1)}$$

Now, since the two fractions have the same denominator, we can add them together by adding the numerators.

$$= \frac{x-1+x+1}{(x-1)(x+1)} = \frac{2x}{(x-1)(x+1)}$$

(2) $\left(\dfrac{-2r}{s}\right)\left(\dfrac{s^2}{-6t}\right)$

Multiplying two fractions is easy: just multiply the numerators and multiply the denominators. So we get:

$$= \frac{-2rs^2}{-6st}$$

Here the numerator and denominator both contain a factor of $-2s$, which we can cancel out. This leaves

$$= \frac{rs}{3t}.$$

Solve.

(3) $2x^2 + 7x = 4$

In general, the only way to solve a polynomial is to make it equal zero, so the first thing we need to do is subtract 4 from both sides.

$$2x^2 + 7x - 4 = 0$$

Now we try to factor, if we can. The only possible factors of $2x^2$ to use are $2x$ and x, so we are looking to factor the polynomial into an expression like:

$$(\,2x + \_\_)(\,x + \_\_\,) = 0$$

We can factor -4 as either -4 times 1, or 2 times -2, or 4 times -1. Checking, we see that

$$(\,2x - 1\,)(\,x + 4\,) = 0$$

is the right factorization. Therefore, the solutions are when either $2x - 1 = 0$ or $x + 4 = 0$. Solving for x, we get two solutions:

$$x = \frac{1}{2} \text{ and } x = -4.$$

Note: You could also use the quadratic equation $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ to solve this problem, but only after subtracting 4 from both sides.

(4) $x^3 + 2x^2 + x = 0$

This time every term is divisible by x, so we factor out an x. This gives us:

$$x(x^2 + 2x + 1) = 0$$

The quadratic polynomial inside the parentheses factors too:

$$x(x + 1)(x + 1) = 0$$

So the solutions are

$$x = 0 \text{ and } x = -1.$$

Simplify the expression.

(5) $\dfrac{\sqrt[4]{32x^4}}{\sqrt[4]{2}}$

Remember that the fourth root is really the same as the $\frac{1}{4}$-power. So this expression can be rewritten

$$= \frac{(32\,x^4)^{1/4}}{2^{1/4}}$$

Now both the 32 and the x^4 are being raised to the $\frac{1}{4}$-power, so be sure to distribute the power to both when you simplify:

$$= \frac{32^{1/4}(x^4)^{1/4}}{2^{1/4}}$$

Now $(x^4)^{1/4} = x$, since you multiply the powers (actually, if x is negative, then $(x^4)^{1/4} = |x|$. However, usually when we work with fractional powers, we assume that x is non-negative so that we don't have to worry about this). We also know that, $32 = 2^5$. Therefore $32^{1/4} = 2^{5/4}$. Making these two simplifications give us this:

$$= \frac{2^{5/4}x}{2^{1/4}}$$

Then we have $2^{5/4}$ in the numerator and $2^{1/4}$ in the denominator so we can subtract their powers ($2^{5/4} \div 2^{1/4} = 2^{5/4-1/4}$).

$$= 2^{5/4-1/4}x = 2x.$$

(6) $(4x^2y^4)^{3/2}$

Each factor in the parentheses gets a power of 3/2. So we have:

$$4^{3/2}(x^2)^{3/2}(y^4)^{3/2}$$

Now remember that $(x^2)^{3/2} = x^{(2)(3/2)} = x^3$ and $(y^4)^{3/2} = y^{(4)(3/2)} = y^6$. Finally, $4^{3/2} = 2^3 = 8$. So our final answer is

$$= 8x^3y^6.$$

(7) $\dfrac{15/4}{5/8}$

To divide two fractions, multiply the top fraction by the reciprocal of the bottom fraction (flip and multiply):

$$= \left(\frac{15}{4}\right)\left(\frac{8}{5}\right) = \frac{(15)(8)}{(4)(5)}$$

Now the both the numerator and the denominator contain the factors 5 and 4, so we cancel those out and we are left with

$$= \frac{(3)(2)}{(1)(1)} = 6.$$

(8) $\dfrac{x^3y^{-1}}{x^{-2}y^{-3/2}}$

Since the x and y factors are being **multiplied**, we can split this fraction into two fractions so that x and y are separated.

$$\frac{x^3y^{-1}}{x^{-2}y^{-3/2}} = \left(\frac{x^3}{x^{-2}}\right)\left(\frac{y^{-1}}{y^{-3/2}}\right)$$

Now, remember that in a fraction with different powers of a variable on the top and bottom, you subtract the powers.

$$\left(\frac{x^3}{x^{-2}}\right) = x^{3-(-2)} = x^5$$

and

$$\left(\frac{y^{-1}}{y^{-3/2}}\right) = y^{-1-(-3/2)} = y^{1/2}$$

Now we multiply the two parts together to get the final answer

$$= x^5y^{1/2}.$$

Rationalize the denominator.

(9) $\dfrac{3}{5-\sqrt{2}}$

To rationalize means to get rid of the square root. In order to do this, one trick is to multiply $5-\sqrt{2}$ by its conjugate: $5+\sqrt{2}$. Of course, we don't want to change the value of the expression when we do this, so we multiply both the top and the bottom of the fraction by the conjugate.

$$= \frac{3}{5-\sqrt{2}}\frac{(5+\sqrt{2})}{(5+\sqrt{2})}$$

When we multiply $(5-\sqrt{2})(5+\sqrt{2})$ in the denominator we get $25+5\sqrt{2}-5\sqrt{2}-2$ and the cross terms, $+5\sqrt{2}$ and $-5\sqrt{2}$, cancel each other out (what remains is the difference of two squares: $(5)^2-(\sqrt{2})^2$). So we have:

$$= \frac{15+3\sqrt{2}}{23}.$$

Solve the inequality in terms of intervals and illustrate the solution set on the real number line.

(10) $x^2 - 2x - 8 < 0$

To solve a polynomial inequality, you always have to factor the polynomial first. This one becomes

$$(x-4)(x+2) < 0.$$

In order for this equation to be true one of the two factors must be positive and the other has to be negative. Since $(x-4)$ is smaller than $(x+2)$, $(x-4)$ will have to be negative and $x+2$ will have to be positive. Thus

$$x - 4 < 0 \text{ and } x + 2 > 0$$

and we can solve both equations simultatenously

$$x < 4 \text{ and } x > -2.$$

(11) $|x+5| \geq 2$

To solve an inequality with an absolute value, you have to get rid of the absolute value first. When you get rid of the absolute value, there are two possibilies. Either $x+5$ was already positive, and then we get $x+5 \geq 2$. On the otherhand, $x+5$ might have been negative, in which case $x+5 \leq -2$. We can now solve these two inequalities at the same time.

$$x + 5 \geq 2 \text{ or } x + 5 \leq -2$$

and we subtract five from all sides to get the final answer

$$x \geq -3 \text{ or } x \leq -7.$$

For the following problem answer part (a) and (b).

(12) As dry air moves upwards, it expands and in so doing cools at a rate of about 1°C for each 100-m rise, up to about 12 km.

(a) If the ground temperature is 20°C, write a formula for the temperature at height h.
This is a linear relationship, so the formula will be the formula for a line. We can use

$$y = mx + b$$

where the y-variable is temperature (call it T) and the independent x-variable is h. Note that T equals 20°C when $h = 0$, so that is our value for b. Finally we need to find the slope m. Remember that slope is rise over run,

$$m = \frac{\Delta y}{\Delta x} = \frac{\Delta T}{\Delta h} = \frac{-1}{100}$$

Putting everything together, we get a formula

$$T = \frac{-1}{100}h + 20.$$

(b) What range of temperature can be expected if a plane takes off and reaches a maximum height of 5 km? The temperature on the ground is the warmest at 20°C and the temperature at 5 km will be the coldest. We can find the temperature at 5 km using the formula from part (a). Be careful, since height was measured in meters in part (a), not kilometers. We will need to change 5 km into 5000 meters to use the formula. Then

$$T = \frac{-1}{100}(5000) + 20 = -30$$

So the range of temperature will be from 20°C down to −30°C.