Calculus I — Math 141 Midterm I

Name: _____

Instructions: You must show all work to earn full credit. No calculators allowed. If you do not have room in the given space to answer a question, use the back of the formula sheet and *indicate clearly* which work goes with which problem.

| Problem | Maximum Points | Your Points |
|---|---|---|
| 1 | 16 | |
| 2 | 8 | |
| 3 | 8 | |
| 4 | 16 | |
| 5 | 18 | |
| 6 | 12 | |
| 7 | 8 | |
| 8 | 8 | |
| Total | 100 | |

1. (16 points) Find the following limits, if they exist. If the limit does not exist, explain why.

   (a) $\lim_{x \to 2} \dfrac{|x-2|}{x-2}$

   (b) $\lim_{x \to 1} \dfrac{x^2+2x-3}{x^2+4x-5}$

2. (8 points) Use the *definition of the derivative* to find $f'(x)$ for $f(x) = \sqrt{1+x}$

3. (8 points) Explain why $x - \cos x = 0$ must have a solution on the interval $[0, \pi/2]$.

4. (16 points) A cylindrical tank holds 90 gallons of water. Water is slowly leaking out of a hole at the bottom. Suppose that the amount of water in the tank at time $t$ hours is

$$V(t) = 90\left(1 - \frac{1}{3}t\right)^2$$

(a) When is the tank is completely empty?

(b) What does the derivative $V'(t)$ represent? What units are used to measure $V'(t)$? Write a sentence explaning your answer. Note: you do not have to calculate the derivative.

5. (18 points) Compute the following derivatives.

(a) $\dfrac{d}{dx}\, x^2\sqrt{x}$

(b) $\dfrac{d}{dx}\, x^2 + \sqrt{\pi}$

(c) $\dfrac{d^2}{dx^2}\, x^3 + \cos x$

6. (12 points) A car is travelling 80 ft/s when the driver suddenly applies the brakes. After $t$ seconds the car is $s(t) = 80t - 8t^2$ feet from the point where the brakes were applied.

   (a) Find a formula for the velocity of the car $v(t)$.

   (b) At what time will the car come to a complete stop?

7. (8 points) For what value(s) of the constant $c$ is the function below continuous?

$$f(x) = \begin{cases} x^2 + 1 & x \geq 2 \\ cx + 7 & x < 2 \end{cases}$$

8. (8 points) Let $f(x) = x^3 - 5x + 1$. Find an equation for the tangent line to $f(x)$ when $x = 1$.

# MATH 141 - Midterm 1 Formula Sheet

## Quadratic Formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

## Common Trigonometric Ratios

| $\theta$ | $0$ | $\pi/6$ | $\pi/4$ | $\pi/3$ | $\pi/2$ |
|---|---|---|---|---|---|
| $\cos\theta$ | $1$ | $\sqrt{3}/2$ | $\sqrt{2}/2$ | $1/2$ | $0$ |
| $\sin\theta$ | $0$ | $1/2$ | $\sqrt{2}/2$ | $\sqrt{3}/2$ | $1$ |

## Obscure Trigonometry Ratios

$$\cot\theta = \frac{\cos\theta}{\sin\theta}, \qquad \csc\theta = \frac{1}{\sin\theta}$$

## Special Trigonometry Limits

$$\lim_{x\to 0} \frac{\sin x}{x} = 1, \qquad \lim_{x\to 0} \frac{\cos x - 1}{x} = 0$$