Math 441 - Homework 10 Due Monday, Dec. 7

1. Let f be differentiable on $\mathbb{R}$. Suppose that $f(0) = 0$ and that $1 \leq f'(x) \leq 2$ for all $x \geq 0$. Use the Mean Value Theorem to prove that $x \leq f(x) \leq 2x$ for all $x \geq 0$.

2. Prove that e is irrational. (Use the outline given in 28.13)

3. Let f be continuous on $[a, b]$ and suppose that $f(x) \geq 0$ for all $x \in [a, b]$. Prove that if the lower integral $L(f) = 0$, then $f(x) = 0$ for all $x \in [a, b]$.