**Math 441 - Homework 4** Due Wednesday, Sept. 23

1. Let $S \subseteq \mathbb{R}$ and let $\bar{S}$ denote the intersection of all the closed sets that contain $S$. Prove that $\bar{S} = \operatorname{cl} S$. Hint: You can prove that $\bar{S}$ is the smallest closed set containing $S$ by showing that any closed set $C$ containing $S$ also contains $\bar{S}$.

2. Prove that any infinite subset of a compact set $K$ has an accumulation point in $K$.

   (Hint: *If no point is an accumulation point, then every point has a neighborhood around it that is disjoint from the other points... can you use this with the definition of compactness?*)

3. Prove the converse to problem #3. That is, prove that for any set $S \subseteq \mathbb{R}$, if every infinite subset of $S$ has an accumulation point in $S$, then $S$ is compact.

   (Hint: *Sometimes it is easier to use the definition of compactness, like in problem #2. Usually it is easier to prove that something is compact by proving that it is closed and bounded.* )