Math 441 - Homework 6 Due Wednesday, Oct. 21

1. Suppose that (s_n) is a convergent sequence with $a \leq s_n \leq b$ for all $n \in \mathbb{N}$. Prove that $a \leq \lim s_n \leq b$.

2. Prove that every Cauchy sequence is bounded. (Note: *this is Lemma 18.11, so you cannot use Theorem 18.12 to prove this*).

3. Prove that every sequence has a monotone subsequence. (Hint: *You might need two cases: one if you are dealing with a sequence that is already monotone, another if you are dealing with a sequence that is not monotone.*)