Math 441 - Homework 9

1. Suppose $f : \mathbb{R} \to \mathbb{R}$ is an even function (i.e., $f(x) = f(-x)$ for all $x \in \mathbb{R}$).

   (a) Prove that if f is differentiable at 0, then $f'(0) = 0$.

   (b) Prove that the conclusion of part (a) is false if f is not differentiable at 0.

   (c) For extra credit, what can you say about the higher derivatives of f at 0? Try some examples, then make a conjecture and prove it. Be sure that you clearly state your conjecture and clearly explain how you proved it.

2. A function f is said to be **increasing** on an interval I if $x_1 < x_2$ implies that $f(x_1) \le f(x_2)$. Use the Mean Value Theorem to prove that f is increasing if and only if $f'(x) > 0$ for all $x \in I$.