Midterm II Review

The following theorems have names, so you should know them.

- **Bolzano-Weierstrass Theorem**: *Bounded sequences have convergent subsequences.*
- **Monotone Convergence Theorem**: *Every bounded monotone sequence converges.*
- **Cauchy Covergence Criterion**: *All Cauchy sequences in* $\mathbb{R}$ *converge. (Not true in other metric spaces.)*
- **Intermediate Value Theorem**: *If* $f : [a, b] \to \mathbb{R}$ *is continuous and* y *is between* $f(a)$ *and* $f(b)$*, then there exists* $c \in [a, b]$ *such that* $f(c) = y$*.*

Some review problems.

1. Define a sequence.

2. Give an example of a Cauchy series in $\mathbb{Q}$ that does not converge to a point in $\mathbb{Q}$.

3. Suppose that Alice and Bob are playing a game (like chess). What is the negation of the following logical statement: *There exists a move that Alice can make, such that for all moves that Bob can make, there is a move that Alice can make that wins the game.*

4. (True or False) Every bounded sequence is convergent.

5. (True or False) Every monotone sequence is convergent.

6. (True or False) Every sequence has a convergent subsequence.

7. (True or False) Every bounded sequence has a Cauchy subsequence.

8. (True or False) For all sequences (s_n) and (t_n), $\limsup(s_n + t_n) = \limsup s_n + \limsup t_n$.

9. (True or False) The function

$$f(x) = \begin{cases} \sin(\frac{1}{x}) & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$$

is continuous.

10. (True or False) If $f : D \to \mathbb{R}$ is continuous and bounded on D, then D is open.