## Algebraic Structures

## Things You Should Know

- You should know the following definitions for the following: *Equivalence relation, group, subgroup, abelian group, cyclic group, generator, order of a group, order of an element, index of a subgroup, cosets.*
- Here are some of the theorems that you should definitely know: The Division Algorithm, Fundamental Theorem of Arithmetic, Fundamental Theorem of Cyclic Groups, Cayley's Theorem, Theorem of Lagrange
- You should know the following groups:  $\mathbb{Z}$ ,  $\mathbb{R}$ ,  $\mathbb{Q}$ ,  $\mathbb{C}^*$ ,  $\mathbb{R}^*$ ,  $\mathbb{Z}_n$ ,  $S_n$ ,  $A_n$ ,  $D_n$ ,  $K_4$  (Klein 4-group),  $\operatorname{GL}(\mathbb{R}, n)$ ,  $\operatorname{SL}(\mathbb{R}, n)$ ,  $\mathbb{Z}_n^*$ .
- Make sure you review all old homework problems, particularly the short answer problems.

## Example Proof Problems

- 1. Prove that  $1 + 4 + 9 + ... + n^2 = \frac{n(n+1)(2n+1)}{6}$ .
- 2. Prove that  $A \setminus (B \cup C) = (A \setminus B) \cap (A \setminus C)$ .
- 3. Let  $S = \mathbb{R} \setminus \{-1\}$  and define a binary operation on S by a \* b = a + b + ab. Prove that (S, \*) is an abelian group.
- 4. Prove that if H and K are both subgroups of a group G, then  $H \cap K$  is also a subgroup of G.
- 5. Prove that every finite group of even order has a subgroup of order 2.