

Name: _____

Final Presentation Topics

Algebraic Structures

The final project is an in class presentation. You will each have 20 minutes to present a topic selected from one of the following options.

1. Ruler and Compass Constructions

- (a) What is a ruler and compass construction?
- (b) Show how to use a ruler and compass to find the midpoint of a line segment, to find the bisector of an angle, and to construct an equilateral triangle.
- (c) Are there any regular polygons that can't be constructed using a ruler and compass?
- (d) Read section 20.3 in the textbook. Describe the constructible numbers and explain why they form a field.
- (e) Conclude your presentation by describing two impossible constructions (in addition to the impossible construction in part (c)).

2. Extension Fields and Transcendental Numbers

- (a) Read section 20.1 in the textbook. Define an extension field and state Kronecker's Theorem (Theorem 20.1). How is Kronecker's theorem related to the Fundamental Theorem of Algebra?
- (b) Define the terms *algebraic number* and *transcendental number*.
- (c) One proof that transcendental numbers exist uses a counting argument. Sketch this proof by answering the following questions.
 - i. How many real numbers are there?
 - ii. How many polynomials with integer coefficients are there?
 - iii. How many algebraic numbers are there?
- (d) Give three famous examples of transcendental numbers, and for each example find out when the example was proven to be transcendental.
- (e) Try to give a time-line for the major events you have talked about. Tell us who proved each result and when they did it.

3. Wallpaper Groups

- (a) Describe the Euclidean group $E(2)$ and the orthogonal group $O(2)$. Which group is larger?
- (b) Read section 11.2 on symmetry.
- (c) Explain why any element $(A, x) \in E(2)$ with $x \neq 0$ has infinite order, and explain what properties A must satisfy if (A, x) is to have finite order.

- (d) Present Theorem 11.7 and Table 11.1. Be sure to explain all of the terms such as: *point group*, *space group*, *glide reflections*.

4. Sylow Theorems

- (a) Read section 14.1 in the book.
- (b) Define the *center* $Z(G)$ and the *centralizer subgroups* $C(x_i)$.
- (c) Prove Cauchy's theorem (Theorem 14.1).
- (d) Describe the three Sylow theorems.

5. Solution of the Cubic

- (a) Find a good source (I recommend *Journey Through Genius* by William Dunham, which is available in both the HSC and Longwood Libraries).
- (b) What is a depressed cubic equation? How to you reduce a regular cubic equation to a depressed cubic?
- (c) Describe Cardano's method for solving a depressed cubic.
- (d) Can all polynomial equations be solved using techniques like Cardano's? Explain which ones can and cannot, and explain when this problem was solved for different kinds of polynomial equations.